NAVIGATION

PART I.

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NAVIGATION

AND

NAUTICAL ASTRONOMY.

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NAVIGATION

AND

NAUTICAL ASTRONOMY.

PART I.

CONTAINING

RULES FOR FINDING THE LATITUDE AND LONGITUDE, AND THE VARIATION OF THE COMPASS.

With numerous Examples.

By H. W. JEANS, F.R.A.S.

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Author of a Work on "Plane and Spherical Trigonometry;" "Handbook of the Stars;" "Problems in Astronomy, Navigation, &c. with Solutions:" formerly Mathematical Master in the Royal Military Academy, Woolwich; and an Examiner of Officers in the Merchant-Service in Nautical Astronomy, &c.

LONDON:
LONGMAN, BROWN, GREEN, AND LONGMANS.
1858.

PREFACE.

This volume of practical rules and examples will be followed by a theoretical treatise on Navigation and Nautical Astronomy, to be called Part II. It will be the aim of the author, in the forthcoming work, to investigate some of the principal rules and corrections in Nautical Astronomy; but in order to simplify the subject as much as possible, Part II. will be made to consist only of such investigations as require in the student nothing beyond a knowledge of Trigonometry: and only the most simple of these investigations will be selected. Therefore the demonstrations of the rules for finding the longitude by occultations, by solar eclipses, transits, &c., as well as the explanation of refraction, of precession, interpolation, &c., and of certain methods for determining the latitude and longitude, which from the peculiar elegance of the artifices used are instructive, and deserve the consideration of the student, —not being of general use, will in Part II. be omitted. They may hereafter be collected and formed into a separate volume, as Part III.

To the present volume is added a rule for finding the time of high-water: it has been now introduced into the book chiefly for students who use Inman's Tables only.

No rule is given in this volume for finding the latitude by pole-star; the one in the *Nautical Almanac*, a work which every naval student is supposed to possess, being deemed suf-

ficient. Examples are given for finding the latitude by the altitude of the pole-star in the College Examination-Papers at the end of the volume.

Another important and very necessary addition will be found in the page of errata. To obtain this, the examples have been recalculated, and the author trusts the principal errata have been discovered: still, in a work like the present, in which so many thousand figures must be used to obtain the answers, it is difficult to avoid mistakes; and it is only after several revisions that a near approach to correctness can be made.

ROYAL NAVAL COLLEGE,
October 21, 1857.

CORRECTIONS.

Note: *t* signifies from the top, and *b* from the bottom.

PAGE	LINE					
12	21	*t* . .	*alter*	6120	*to*	6080
15	9	*b* . .	*alter* at *to* of: *it should read* bearing of, *instead*			
„	11		*of* bearing at.			
46	4	*t* . .	*alter*	S.S.W.	*to*	S.S.W.½W.
56	5	*t* . .	„	equator	„	concave.
68	2	*b* . .	„	$5^h\ 38^m\ 38^s$	„	$5^h\ 38^m\ 50^s$
69	1	*t* . .	„	$7^h\ 13^m\ 55{\cdot}4^s$	„	$7^h\ 13^m\ 37{\cdot}4^s$
„	4	*t* . .	„	230°	„	240°
„	1	*b* . .	„	227	„	257
71	9	*t* . .	„	$8^h\ 25^m$	„	$8^h\ 26^m$
78	5	*b* . .	„	$5^m\ 3{\cdot}7^s$	„	$4^m\ 56{\cdot}0^s$
79	19	*t* . .	„	48° 42′	„	56° 15′
81	1	*b* . .	„	$0^m\ 10{\cdot}0^s$	„	$1^m\ 40{\cdot}0^s$
84	4	*t* . .	„	2^h 42m	„	$2^h\ 32^m$
„	11	*b* . .	„	15	„	16
85	5	*b* . .	„	70°	„	74°
86	7	*t* . .	„	17	„	7
87	11	*b* . .	„	52″	„	42″
91	1	*b* . .	„	$18^h\ 5^m\ 18{\cdot}8^s$	„	$18^h\ 5^m\ 10{\cdot}8^s$
99	3	*b* . .	„	east	„	west.
108	6	*t* . .	*after* correction, *add* +2′ 42″ and height of eye above the sea.			
113	5	*b* . .	*alter*	latitudes	*to*	altitudes.
116	13	*b* . .	„	meridian	„	equator.
118	18	*t* . .	the correction 4′ 38″ should be *added:* the correct latitude will then be 54° 53′ 21″ N.			

PAGE	LINE					
121	1	*b* . .	*alter*	lower	*to*	upper.
128	3	*t* . .	„	47	„	37
143	2	*b* . .	„	28·8	„	20·8
150	10	*t* . .	„	April 24	„	October 24.
181	10	*t* . .	„	4″·5	„	45″
207	11 }	*b* . .	„ moon *into* sun, *and* sun *into* moon.			
„	12 }					
214	15	*t* . .	„	133	*to*	123
219	20	*t* . .	„	54	„	56
220	6	*t* . .	„	5	„	23° 3′ 4″

224 Make the following addition to the example worked out:

App. Dist. . .	72° 42′ 16″
Aux. A. . . .	60 15 21
Sum	132 57 37 vers.
Difference . . .	12 26 55 vers.

230	1	*t* . .	*alter*	stars	*to*	sun.
232	4	*t* . .	„	1′ 55″	„	0″

237 In the example worked out the horizontal parallax should be 7″ less.

238	8	*b* . .	*alter*	lat.	*to*	alt.
245	3	*b* . .	„	32′	„	39′
246	2	*b* . .	„	23½	„	20½
249	9	*b* . .	„	67° 14′ 45″	„	14° 55′
254	7	*t* . .	„	50° 48′	„	50° 48′ N.
258	9	*t* . .	„	P.M.	„	A.M.

205 The analytical expression in this page is incorrectly printed: it will be found correct in its proper place in Part II.

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Variation of Compass.

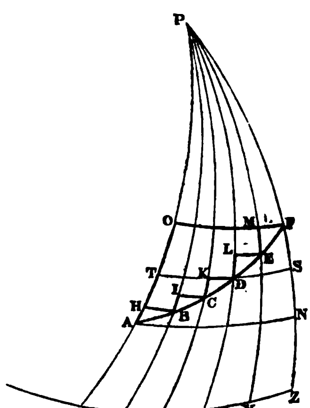
NAVIGATION.

NAVIGATION.

DEFINITIONS, ETC.

(1.) Two distinct methods are used for navigating a ship from one place to another; the first is an application of the common rules of plane trigonometry; the other requires a knowledge of spherical trigonometry, and of the principal definitions and facts in astronomy. The latter is for this reason called *Nautical Astronomy;* the characteristic name of the former being *Navigation* or *plane sailing*.

(2.) The necessary angles and measurements in the first method are supplied by means of the compass and log-line; in the second and more exact method they are obtained by astronomical observations.



Definitions in Navigation.

Let A and F represent two places on the surface of the earth (considered as a sphere), PU, PZ, their meridians, P the pole, and UZ an arc of the equator. Through A and F draw a curve line AF, cutting all the intermediate meridians, PV, PW, &c., *at the same angle.* This common angle is called the *course* from A to F, and the arc AF (in nautical miles) is

called the *distance*. Draw the parallels of latitude AN and FO; the arc AU is the latitude of A, and FZ the latitude of F; UZ, or the angle APF, is the difference of longitude between A and F. The arc OA is the difference, or, as it is called in Navigation, the *true difference of latitude* between A and F.

Suppose the intermediate meridians PV, PW, &c. to be drawn through points B, C, &c. taken on the arc AF *indefinitely near to one another*, and through B, C, &c., suppose arcs of parallels BH, CI, &c. to be drawn; on this supposition the elementary triangles ABH, BCI, &c. may be considered as *right-angled plane triangles*, of which the sum of the sides AB, BC, &c. is the distance, the sum of the sides AH, BI, &c. is equal to the true difference of latitude, and the sum of the sides BH, CI, &c. is called in Navigation the *departure*.

The chart used at sea for marking down the ship's track, and for other purposes, exhibits the surface of the globe on a plane on which the meridians are drawn parallel to each other, and therefore the parts BH, CI, DK, &c., arcs of parallels of latitude, are increased and become equal to the corresponding parts of the equator UV, VW, &c. Now, in order that every point on this plane may occupy the same relative position with respect to each other that the points corresponding to them do on the surface of the globe, the distance between any points A and O, and A and F must be increased in the same proportion as the distance FO has been increased. The true difference of latitude, AO, is thus projected on the chart into what is called the *meridional difference* of latitude, and the departure, BH+CI+DK+ . . . into the difference of longitude. A chart constructed in this manner is called a Mercator's Chart. From these definitions and principles are deduced certain trigonometrical formulæ, and these expressed in words form the common Rules of Mercator and Parallel Sailing. For the proof of these formulæ and rules, the student is referred to the author's volume of "Astronomical Problems and their Solutions," (p. 122.)

PRELIMINARY RULES IN NAVIGATION.

Rule (a).

To find the true difference of latitude, having given the latitude from and latitude in.*

(1.) When latitude from and latitude in have *like names*, that is, are both north or both south.

Under the latitude from, put down the latitude in, take the difference and reduce the same to minutes; place N. or S. against the result according as the latitude in is north or south of the latitude from; the remainder is the *true difference of latitude.*

(2.) When latitude from and latitude in have *unlike names*, that is, one north and the other south.

Take the sum of the two latitudes, reduce it to minutes, and attach N. or S. thereto according as the latitude in is north or south of the latitude from; the result is the *true difference of latitude.*

EXAMPLES.

1. Find the true difference of latitude, having given latitude from = 42° 10′ N., and latitude in 50° 48′ N.

lat. from	42° 10′	N.
lat. in	50 48	N.
	8 38	
	60	
T. D. lat.	518	N.

* The latitude of the place left is called the latitude *from*; the latitude of the place arrived at is called the latitude *in*.

2. Find the true difference of latitude, having given latitude from 3° 42′ N., and latitude in 2° 50′ S.

lat. from	3° 42′ N.
lat. in	2 50 S.
	6 32
	60
T. D. lat.	392 S.

Find the true difference of latitude in each of the following examples:

	Lat. from.	Lat. in.	Answers.
3.	33° 42′ N.	40° 40′ N.	T. D. lat. = 418 N.
4.	40 40 N.	33 42 N.	... = 418 S.
5.	3 42 S.	1 40 N.	... = 322 N.
6.	3 8 S.	14 42 S.	... = 694 S.
7.	68 48 N.	38 30 N.	... = 1818 S.
8.	14 14 N.	0 0	... = 854 S.

Rule (b).

To find the meridional difference of latitude, having given the latitude from and latitude in.

Take the meridional parts for the two latitudes from the table of meridional parts; subtract, if the names be alike, and add if the names be unlike; the result is the *meridional difference of latitude*, N. or S. being attached thereto according as the latitude in is north or south of latitude from.

EXAMPLES.

9. Find the meridional difference of latitude, having given latitude from 42° 10′ N., and latitude in 50° 48′ N.

lat. from	42° 10′ N.	mer. parts	2795·2 N.
lat. in	50 48 N.	mer. parts	3549·8 N.
		mer. diff. lat.	754·6 N.

10. Find the meridional difference of latitude, having given latitude from 3° 42′ N., and latitude in 7° 32′ S.

lat. from	3° 42′ N.	mer. parts	222·2 N.
lat. in	7 32 S.	mer. parts	453·3 S.
		mer. diff. lat.	675·5 S.

Find the meridional difference of latitude in each of the following examples:

	Lat. from.	Lat. in.	Answers.
11.	34° 42′ N.	33° 15′ N.	M. D. lat. = 104·9 S.
12.	14 14 N.	30 14 N.	... = 1041·7 N.
13.	84 10 N.	80 30 N.	... = 1681·5 S.
14.	2 8 S.	3 10 N.	... = 318·1 N.
15.	4 5 N.	4 5 S.	... = 490·4 S.
16.	0 0	2 45 N.	... = 165·1 N.

Rule (*c*).

To find the middle latitude, having given the latitude from and latitude in.

The names being supposed to be *alike*, that is, both north or both south.

Add together the two latitudes, and take half the sum; the result is the middle latitude.

When the names are unlike, the mid. lat. (which is seldom required but for obtaining the departure) should be found by means of a table; but in this case it may perhaps be as well to avoid the use of the middle latitude in any of the common problems in navigation.

EXAMPLES.

17. Find the middle latitude, having given latitude from 3° 42′ N., and latitude in 13° 52′ N.

lat. from	3° 42′	N.
lat. in	13 52	N.
	2)17 34	
mid. lat.	8 47	N.

Find the middle latitude in each of the following examples:

	Lat. from.	Lat. in.	Answers.
18.	38° 42′ N.	30° 30′ N.	mid. lat. 34° 36′ N.
19.	62 17 S.	62 30 S.	... 62 23½ S.

Rule (d).

To find the difference of longitude, having given the longitude from and longitude in.

(1.) When the longitude from and longitude in have *like names;* that is, are both east or both west.

Under longitude from put longitude in, take the difference, and reduce the same to minutes; place E. or W. against the remainder according as the longitude in is east or west of longitude from; the remainder will be the difference of longitude.

(2.) When the longitude from and longitude in have *unlike names,* that is, one east and the other west.

Take the sum of the two longitudes, reduce it to minutes, and attach E. or W. thereto according as the longitude in is east or west of the longitude from; the result is the true difference of longitude.

NOTE.—If the difference of longitude found by this rule exceed 180°, it must be subtracted from 360°, and the remainder brought into minutes must be considered the difference of longitude, with the contrary letter attached to it.

EXAMPLES.

20. Find the difference of longitude, having given the longitude from = 110° 42′ W., and longitude in 100° 42′ W.

long. from	110° 42′ W.
long. in	100 42 W.
	10 0
	60
diff. long.	600 E.

21. Find the difference of longitude, having given long. from 12° 10′ E., and long. in 2° 45′ W.

long. from	12° 10′ E.
long. in	2 45 W.
	14 55
	60
diff. long.	895 W.

Find the difference of longitude in each of the following examples;

	Long. from.	Long. in.	Answers.
22.	33° 40′ E.	40° 10′ E.	Diff. long. 390 E.
23.	104 0 W.	110 30 W.	... 390 W.
24.	2 45 W.	3 30 E.	... 375 E.
25.	0 0	4 10 W.	... 250 W.
26.	3 10 W.	3 10 E.	... 380 E.
27.	179 0 E.	179 0 W.	... 120 E.

Rule (*e*).

To find the latitude in, having given the latitude from and true difference of latitude.

(1.) When the latitude from and true difference of latitude have *like names*.

To the latitude from, *add* the true difference of latitude (turned into degrees and minutes, if necessary); the sum will be the latitude in, of the same name as the latitude from.

(2.) When the latitude from and true difference of latitude have *unlike names*.

Under the latitude from put the true difference of latitude (in degrees and minutes, if necessary): take the less from the greater; the remainder, marked with the name of the greater, is the latitude in.

EXAMPLES.

28. Find the latitude in, having given the latitude from 42° 30′ N., and true difference of latitude 342′ N.

60)342′ N.	lat. from	42° 30′ N.	
5° 42′ N.	T. D. lat.	5 42 N.	
	lat. in	48 12 N.	

29. Find the latitude in, having given the latitude from 2° 40′ S., and true difference latitude 342′ N.

60)342′ N.	lat. from	2° 40′ S.
5° 42′ N.	T. D. lat.	5 42 N.
	lat. in	3 2 N.

Find the latitude in, in each of the following examples:

	Lat. from.	T. D. lat.	Answers.
30.	30° 10′ N.	182′ N.	Lat. in 33° 12′ N.
31.	3 2 S.	190 N.	... 0 8 N.
32.	2 48 S.	368 N.	... 3 20 N.
33.	2 48 S.	288 N.	... 2 0 N.
34.	4 48 N.	288 S.	... 0 0
35.	0 10 N.	228 N.	... 3 58 N.

Rule (f).

To find the longitude in, having given the longitude from and the difference of longitude.

(1.) When the longitude from and diff. long. have *like names.*

To the long. from, *add* diff. long. (turned into degrees if necessary); the sum will be long. in, of the same name as long. from.

(2.) When the long. from and diff. long. have *unlike names.*

Under long. from, put diff. long. (in degrees and minutes, if necessary); take the less from the greater; the remainder, marked with the name of the greater, is the long. in.

NOTE.—If the long. in, found as above, exceed 180°, subtract it from 360°, and attach to the remainder the contrary name to the one directed in the Rule.

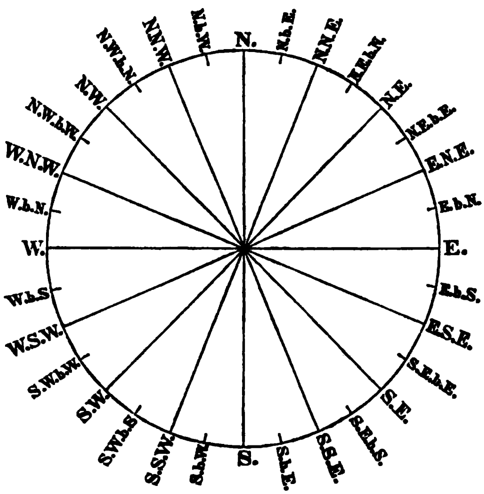
EXAMPLES.

36. Find the long. in, having given long. from 38° 42′ W., and diff. long. 384·5′ W.

60)384·5	long. from	38° 42′	W.
6° 24·5′ W.	diff. long.	6 24·5	W.
	long. in	45 6·5	W.

Find the longitude in, in each of the following examples:

	Long. from.	Diff. long.	Answers.
37.	62° 32′ E.	1000·5′ W.	long. in 45° 51·5′ E.
38.	2 30 E.	126·6 E.	... 4 36·6 E.
39.	3 40 W.	220·0 E.	... 0 0
40.	0 0	100·4 W.	... 1 40·4 W.
41.	179 59 W.	2·0 W.	... 179 59·0 E.



THE COMPASS AND LOG-LINE.

The Compass card is represented above; each quadrant is divided into eight equal parts, called *points;* each point therefore contains 11° 15′. The names of the points are formed as follows:

Middle point between N. and E. is formed by putting these letters together, thus . . . . . N.E.
Middle point between N. and N.E. is formed thus N.N.E.
Middle point between E. and N.E. is formed thus E.N.E.
One point from N. towards E. is N. by E. or . . N.b.E.
... E. ... N. is E. by N. or . . E.b.N.
... N.E. ... N. is N.E. by N. or N.E.b.N.
... N.E. ... E. is N.E. by E. or N.E.b.E.

The points of the compass are frequently referred to with respect to their position to the right or left of the cardinal point towards which the spectator is looking; thus, N.E. is

said to be 4 points to the right of N; E.b.N. is 7 points to the right of north. Each point, moreover, is subdivided into quarter points, and named from the adjacent points; thus, $2\frac{1}{2}$ points to the right of north is N.N.E.$\frac{1}{2}$E.; $7\frac{3}{4}$ points to the right of north is E.b.N.$\frac{3}{4}$E. or E.$\frac{1}{4}$N.

The other three quadrants are divided and referred to in a similar manner.

Attached to the compass, and coinciding with the line N. S., is a magnetic bar of steel, by means of which the card, when balanced on a fine point near its centre, will indicate the compass bearing or direction of any object beyond it.

Thus, the compass being placed near the helm, the bearing of the ship's head is seen at once, and the direction in which the ship is steered is readily noted.

The Log-line.

The log is a flat piece of thin wood of a quadrantal form, loaded in the circular side with lead sufficient to make it swim upright in the water; to this is fastened a line about 150 fathoms long, called the *log-line*, which is divided into certain spaces called *knots;* the length of each knot is supposed to be the same part of a nautical mile (6120 feet) that half a minute is of an hour, hence 1 knot $= \frac{6120}{120} = 51$ feet. If, therefore, 1 knot runs out in half a minute (shown by a half-minute glass), the rate of the ship is supposed to be 1 mile an hour; if 2 knots, the rate is 2 miles an hour, and so on. The length of the knot is very rarely so much as 51 feet, and the hour-glass used is not always a half-minute glass; various modifications of the two instruments are made, to render this method of measuring the ship's way tolerably correct; these will be more clearly seen in the use of the instruments themselves.

Correcting Courses.

Three corrections are sometimes necessary to be applied

to the course steered by compass, to reduce it to the true course; and the converse. These are called:

(1.) The variation of the compass.
(2.) The deviation of the compass.
(3.) The leeway.

(1.) *The Variation of the Compass.*

The magnetic needle seldom points to the true north. Its deflection to the east or west of the true north is called the *variation of the compass;* it is different in different places, and it is also subject to a slow change in the same place. The variation of the compass is ascertained at sea by observing the magnetic bearing of the sun when in the horizon, or at a given altitude above it. From this observation the true bearing is found by rules given in nautical astronomy. The difference between the true bearing and the observed bearing by compass determines this correction.

The method of correcting the course for variation will be more readily understood by means of an example.

Suppose the variation of the compass is found to be 2 points to the east, that is, the needle is directed 2 points to the right of the north point of the heavens; then the N.N.W. point of the compass card will evidently point to the true north, and every other point on the card will be shifted round 2 points. If, therefore, a ship is sailing *by compass* N.N.W., or, as it is expressed, the compass course is N.N.W., her true course will be north; that is, 2 *points to the right of the compass course.* In a similar manner it may be shown that, when the variation is 2 points westerly, the true course will be 2 *points to the left of compass course.* Hence this rule:

Rule (*g*).

To find the true course, the compass course being given.

Easterly variation allow to the right.
Westerly ... ... left.

From the preceding considerations it will be easy to deduce the converse rule, namely:

Rule (h).

To find the compass course, the true course being given.

Easterly variation allow to the left.
Westerly ... ... right.

EXAMPLES TO RULES (*g*) AND (*h*).

42. Find the true course, having given the compass course N.W.½W. and variation 3¼ west.

	pts.	qrs.	
Compass course .	4	2	left of N.
variation . . .	3	1	left.*
true course . .	7	3	left of N. = W.¼N.

43. Find the compass course, having given the true course W.¼N. and variation 3¼ W.

	pts.	qrs.	
True course . .	7	3	left of N.
variation . . .	3	1	right.
compass course .	4	2	left of N. = N.W.½W.

Find the true course in each of the following examples:

	Compass course.	Var.	Answers.
44.	N.N.E.	2¼W.	N.¼W.
45.	N.W.	1¾E.	N.N.W.¼W.
46.	S.W.¾W.	1½E.	W.S.W.¼W.
47.	S.	2W.	S.S.E.
48.	W.	2½E.	N.W.b.W.½W.

Find the compass course in each of the following examples:

	True course.	Var.	Answers.
49.	N.N.E.½E.	¼W.	N.N.E.¾E.
50.	N.	1½E.	N.b.W.½W.
51.	S.S.W.	2W.	S.W.
52.	S.W.	0	S.W.
53.	N.b.W.¼W.	1¼W.	N.

* When names are alike, (that is, both left or both right,) *add*: when unlike, *subtract*, marking remainder with the name of the greater.

(2.) *Deviation of the Compass.*

This correction of the compass arises from the effect of the iron on board ship on the magnetic needle, in deflecting it to the right or left of the magnetic meridian. The increased quantity of iron used in ships, especially in steamers, has caused this correction to be attended to now more than formerly, as its effects and magnitude have become more perceptible. The amount of the deviation arising from this local cause varies as the mass of iron changes its position with respect to the compass. When a fore and aft line coincides with the direction of the magnetic meridian, the iron in the ship may be supposed to be nearly equally distributed on both sides of the needle, and its effect in deflecting the needle may be inappreciable. In other positions of the ship with respect to the magnetic meridian, the iron may produce a sensible deflection of the needle; and this deflection or deviation will in general be the greatest when the ship's head points to the east or west.

Various methods are used to determine this correction. The one usually adopted is to place a compass on shore, where it may be beyond the influence of the iron of the ship, or any other local disturbing force, and to take the bearing of the ship's compass, or some object in the same direction therewith; at the same time, the observer on board takes the bearing of the shore compass; then, if 180° be added to the bearing at the shore compass, so as to bring it round to the opposite point, the difference between the result and the bearing at ship's compass will be the amount of the deviation of the compass for that position of the ship. Thus, suppose the following bearings are taken when the direction of the ship's head is N.

	Shore (reading off).		Ship (reading off).
	S. 17° W.		N. 20° E.
	180		N. 17 E.
	S. 197 W.	Dev.	3 E.
or	N. 17 E.		

From this it appears the deviation, when the ship's head is north, is 3° easterly. The ship is then swung round one or two points, and a similar observation made; and thus the local deviation found for a second position of the ship. This being repeated for every point or two points of the compass, the deviation is thus known for all positions of the ship. A table, similar to the one below, is then formed, and the courses corrected for this deviation by the following rules; which resemble those already given for correcting for variation.

Deviation of Compass of H.M.S. ———, for given positions of the Ship's Head.

Direction of ship's head.		Deviation of compass.	nearly	Direction of ship's head.		Deviation of compass.	nearly
N.	E.	2°45′	or ¼ pt.	S.	W.	3° 0′	or ¼ pt.
N.b.E.	E.	4 57	or ½ „	S.b.W.	W.	4 20	or ½ „
N.N.E.	E.	7 30	or ¾ „	S.S.W.	W.	5 0	or ½ „
N.E.b.N.	E.	9 0	or ¾ „	S.W.b.S.	W.	6 7	or ½ „
N.E.	E.	10 0	or ¾ „	S.W.	W.	7 0	or ½ „
N.E.b.E.	E.	10 55	or 1 „	S.W.b.W.	W.	7 27	or ½ „
E.N.E.	E.	10 40	or 1 „	W.S.W.	W.	7 50	or ¾ „
E.b.N.	E.	9 55	or ¾ „	W.b.S.	W.	8 20	or ¾ „
E.	E.	8 50	or ¾ „	W.	W.	8 50	or ¾ „
E.b.S.	E.	7 15	or ½ „	W.b.N.	W.	8 10	or ¾ „
E.S.E.	E.	5 35	or ½ „	W.N.W.	W.	6 50	or ½ „
S.E.b.E.	E.	3 40	or ¼ „	N.W.b.W.	W.	5 40	or ½ „
S.E.	E.	1 50	or ¼ „	N.W.	W.	4 50	or ½ „
S.E.b.S.	E.	0 20	or 0 „	N.W.b.N.	W.	3 20	or ¼ „
S.S.E.	W.	0 56	or 0 „	N.N.W.	W.	1 40	or 0 „
S.b.E.	W.	2 20	or ¼ „	N.b.W.	E.	1 10	or 0 „

Rule (i).

To find the true course, having given the compass course and the deviation.

Easterly deviation allow to the right.
Westerly ... ... left.

EXAMPLES.

54. Correct the compass course W.b.S. for deviation ¾ W. (known from table, above).

	pts.	qrs.	
Compass course . .	7	0	right of S.
deviation	0	3	left.
true course . . .	6	1	right of S.

or W.S.W.¼W.

55. Correct the compass course N.W.½W. for deviation ½W. (from deviation table, p. 16), and also for variation or compass 3¼ W.

		pts.	qrs.	
Compass course . .		4	2	l. N.
deviation . .	0 2 l.			
variation . .	3 1 l.			
		3	3	l.
true course . . .		8	1	l. N.
		16		
or true course. . .		7	3	r. S. = W.¼S.

Find the true course in each of the following examples, by correcting for deviation from table, p. 16, and for variation:

	Compass course.	Var.	Answers.
56.	N.N.E.	2¼W.	N.½E.
57.	N.W.	1¾E.	N.N.W.¾W.
58.	S.W.¾W.	1½E.	S.W.b.W.¾W.
59.	S.	2W.	S.S.E.¼E.
60.	W.	2½E.	W.N.W.¼W.
61.	W.¾N.	1½W.	W.S.W.½W.

Rule (*k*).

To find the compass course, having given the true course and deviation.

Easterly deviation allow to the left.
Westerly ... ... right.

NOTE.—The true course should first be corrected for variation (if any) by Rule (*h*), (which is similar to the above), so as to get a compass course nearly, and then this course for deviation, from table, p. 16.

EXAMPLES.

62. Required the compass course, the true course being W.S.W.¼W., variation 0, and deviation ¾W. (see table.)

	pts.	qrs.	
True course . .	6	1	r. S.
deviation . . .	0	3	r.
compass course .	7	0	r. S., or W.b.S.

63. Required the compass course, the true course being S.W., variation of compass 2¼E., and deviation as in table, p. 16.

	pts.	qrs.	
True course	4	0	r. S.
variation	2	1	l.
compass course nearly .	1	3	r. S., or S.b.W.¾W.
deviation	0	2	r.
compass course . . .	2	1	r. S. = S.S.W.¼W.

Required the compass course in each of the following examples (for deviation, see table, p. 16):

	True course.	Var.	Answers.
64.	N.½E.	2¼W.	N.N.E.
65.	N.N.W.¾W.	1¾E.	N.W.
66.	S.W.b.W.¾W.	1½E.	S. W.¾W.
67.	S.S.E.¼E.	2W.	S.
68.	W.N.W.¼W.	2½E.	W.
69.	W.S.W.½W.	1½W.	W.¾N.

(3.) *Leeway.*

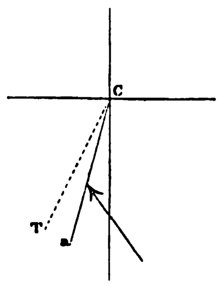
This correction is the angle which the ship's track makes with the direction of a fore and aft line: it arises from the action of the wind on the sails, &c. not only impelling the ship forwards, but pressing against it sideways, so as to cause the actual course made to be to *leeward* of the apparent course, as shown by the fore and aft line. The amount of leeway differs in different ships, depending on their con-

struction, on the sails set, the velocity forwards, and other circumstances. Experience and observation, therefore, usually determine the amount of leeway to be allowed.

Rule (l).

The method of correcting for leeway will be best seen by the following example:

Suppose the apparent course is S.S.W.½W., and leeway 2 points, the wind being S.E., required the correct course.



Draw two lines at right angles to each other towards the cardinal points of compass, and a line, as C *a*, to represent (roughly) the course of the ship, and another to represent the direction of the wind (as the arrow in fig.); then it will be seen that the corrected course, as C T, will be to the *right* of the apparent course; *the observer being always supposed to be at the centre,* C, *and looking towards the cardinal point from whence the course is measured;* hence

	pts.	qrs.	
Apparent course . .	2	2	r. S.
leeway	2	0	r.
corrected course . .	4	2	r. S. = S.W.½W.

EXAMPLES.

Correct the following courses for leeway, so as to find the true courses:

	Apparent course.	Wind.	Leeway.	Answers.
70.	N.N.E.	W.N.W.	1½	N.E.½N.
71.	N.W.	N.N.E.	2	W.N.W.
72.	E.S.E.	S.	2½	E.½N.
73.	E.	N.b.E.	¾	E.¾S.

Correct the following compass courses for deviation, variation, and leeway, so as to find the true courses. The

deviation is found in table, p. 16, and the variation of compass is supposed to be in each example $2\frac{1}{2}$ W.

	Course.	Wind.	Leeway.	Answers.
74.	N.W.$\frac{1}{4}$W.	W.S.W.	$2\frac{1}{2}$	N.W.$\frac{3}{4}$W.
75.	S.E.$\frac{1}{2}$E.	E.N.E.	$2\frac{1}{4}$	S.E.$\frac{1}{2}$E.
76.	W.$\frac{1}{4}$S.	S.S.W.	2	W.S.W.$\frac{1}{2}$W.
77.	N.$\frac{3}{4}$W.	W.b.N.	$1\frac{1}{2}$	N.b.W.$\frac{1}{2}$W.

These examples may be worked out in the following manner:

			qrs.	pts.	
Ex. 74.	Compass course . .		4	1	l. N.
	deviation . .	0 2 l.			
	variation . .	2 2 l.			
			3	0	l.
			7	1	l. N.
	leeway		2	2	r.
	true course . . .		4	3	l. N. = N.W.$\frac{3}{4}$W.

RULES IN NAVIGATION.

Rule 1.

To find the course and distance from one place to another, having given the latitudes and longitudes of the two places.

(1.) Find true difference of latitude, meridional difference of latitude, and difference of longitude: reduce the true difference of latitude and difference of longitude to minutes, attaching thereto the proper letters. Rules (*a*), (*b*), (*d*).

(2.) *To find the course.* From the log difference of longitude (increased by 10), subtract the log. mer. diff. latitude; the remainder is the log. tan. course, which find in the tables, and place before it the letter of the true difference latitude, and after it the letter of the difference longitude,

to indicate the direction of course. At the same opening of the tables, take out the log. secant course.

(3.) *To find distance.* Add together log. secant course, and log. true difference latitude; the sum (rejecting 10 in the index) will be the log. distance, which find in the tables.

EXAMPLES.

78. Required the course and distance from A to B.

lat. A 45° 15′ N.		long. A 35° 26′ W.	
lat. B 47 10 N.	M. P.	long. B 32 15 W.	

lat. A 45° 15′ N.		3051·2 N.	long. A 35° 26′ W.
lat. B 47 10 N.		3217·4 N.	long. B 32 15 W.
1 55	M.D.lat.	166·2 N.	3 11
60			60
T.D. lat. 115 N.			diff. long. 191 E.

log. diff. long. + 10, 12·281033
log. M.D. lat. . . . 2·220631

log. tan. course. . 10·060402 . . . log. sec. course 0·182767
∴ course N. 48° 58′ E. log. T.D. lat. . 2·060698

log. dist. . . . 2·243465
∴ distance 175′

Required also the compass course in the above example: var. of compass being 2 points W, and deviation on account of local attraction, as in table (p. 16). See Rule (*k*).

	pts.	qrs.	
True course . . .	48° 58′ r. N.		
or	4	1	r. N.*
variation	2	0	r.
compass course nearly	6	1	r. N. = E.N.E.¼E.
deviation	1	0	l.
∴ compass course .	5	1	r. N. = N.E.b.E.¼E.

Examples in Navigation are usually worked without attaching to each logarithm taken out its name or designation, as in the following example:

* Degrees are converted into points, or the converse, by means of the table for that purpose in the nautical tables.

Required the course and distance from A to B.

Lat. A 51° 31′ N.			Long. A 0° 6′ W.
B 54 33 N.	M.P.		B 3 5 E.
51° 31′ N.	3618		0° 6′ W.
54 33 N.	3921		3 5 E.
3 2	M.D. lat. 303		3 11
60			60
Diff. lat. 182 N.			Diff. long. 191 E.

12·281033
2·481443
9·799590 . . . . . . 0·072650
N. 32° 13′ 30″ E. 2·260071
Course. 2·332721
215·1 dist.

Required the course and distance from A to B in each of the following examples, by Rule 1, or Mercator's method:

	Lat. from and lat. in.	Long. from and long. in.	Answers. Course and distance.
79.	lat. A 49° 52′ S.	long. A 17° 22′ W.	course N. 26° 36′ E.
	„ B 42 13 S.	„ B 11 50 W.	dist. 513·3 miles.
80.	lat. A 49 10 N.	long. A 29 17 W.	course N. 37° 48′ W.
	„ B 56 45 N.	„ B 39 5 W.	dist. 576 miles.
81.	lat. A 50 48 N.	long. A 1 10 E.	course N. 41° 55′ W.
	„ B 52 35 N.	„ B 1 25 W.	dist. 143·8 miles.
82.	lat. A 58 24 N.	long. A 4 12 W.	course N. 32° 34′ E.
	„ B 63 17 N.	„ B 2 13 E.	dist. 347·6 miles.
83.	lat. A 2 37 N.	long. A 110 42 W.	course S. 75° 11′ W.
	„ B 0 0	„ B 120 36 W.	dist. 614 miles.
84.	lat. A 3 30 N.	long. A 33 40 E.	course S. 42° 31′ E.
	„ B 4 10 S.	„ B 40 42 E.	dist. 624 miles.

Required also the compass courses in examples 82, 83, and 84, the variation of compass being 2 points E., and deviation as in table, p. 16. See Rule (*k*).

ANSWERS.

82. compass course N. ½E. nearly.
83. ... ... S.W.b.W. ¼W. nearly.
84. ... ... E.S.E. ¾E. „

To find the latitude and longitude in, having given the Course and Distance.

Rule 2.

(1). *To find latitude in.* Add together log. cos. course * and log. distance, the sum (rejecting 10 in the index) will be log. true difference latitude, which find in the tables; reduce to degrees and minutes, and place the letter N. or S. against it, according as course is northward or southward.

(2.) Apply true difference latitude to latitude from, so as to get the latitude in. (Rule *e.*)

(3.) *To find longitude in.* Take out the meridional parts for the two latitudes, and get M. D. Lat. (Rule *b.*)

(4.) Add together log. tangent course and log. meridional difference latitude; the sum (rejecting 10 in the index) will be the log. difference longitude, which find in the tables; reduce to degrees and minutes, and place the letter E. or W. against it, according as the course is eastward or westward.

(5.) Apply difference longitude to longitude from, so as to get longitude in. (Rule *f.*)

EXAMPLES.

85. Sailed from A, N. 37° 10′ E., 472·6 miles; required the latitude and longitude in.

lat. A	27° 20′ N.		long. A	25° 12′ E.
log. cos. course	9·901394		log. tan. course	9.879740
log. dist.. . .	2·674494		log. M.D. lat.	2·641474
log. T.D. lat. .	2·575888		log. diff. long.	2·521214
∴ T.D. lat.	376·6′		diff. long.	332·1′
or	6° 17′ N.	M.P.	or	5° 32′ E.
lat. from	27 20 N. . . .	1706 N.	long. from	25 12 E.
lat. in	33 37 N. . . .	2144 N.	long. in	30 44 E.
	M.D. lat.	438		

* Take out, at same opening of tables, log. tan. course and place it a little to the right.

A ship in latitude 27° 0′ S. and longitude 123° W. sailed S.S.E.½E., 150 miles: required the latitude and longitude in.

S.S.E.½E. = 28° 7′ 30″

	9·945430		9·727957
	2·176091		2·176091
	2·121521		1·904048
	6,0)13,2·3		80′.1
Diff. lat.	2° 12′ 18″ S.	M.P.	1° 20′ 6″ E.
Lat. from . . .	27 0 0 S.	1683 S.	123 0 0 W.
Lat in	29 12 18 S.	1833 S.	121 39 54 W.
		150	Long. in.

Required the latitude and longitude in, by Rule 2, or Mercator's method, in each of the following examples, having sailed from A as follows:

	Course and dist. from A.		Lat. A.	Long. A.	Answers. Lat. in.	Long. in.
86.	N. 26° 36′ E.	513·5′	49° 52′ S.	17° 22′ W.	42° 13′ S.	11° 50′ W.
87.	S. 48 58 W.	175·2	47 10 N.	32 15 W.	45 15 N.	35 26 W.
88.	N. 29 10 E.	373·4	52 10 N.	17 32 W.	57 36 N.	12 15 W.
89.	N. 31 4 W.	318·8	57 40 N.	12 16 W.	62 13 N.	17 45 W.
90.	S. 37 7 E.	370·0	70 14 S.	25 30 E.	75 9 S.	38 5 E.
91.	N. 47 47 E.	272·4	50 15 S.	15 10 E.	47 12 S.	20 16 E.

To find the course and distance by Middle Latitude method.

Rule 3.

(1.) Find the true difference latitude, middle latitude, and difference longitude (*a*), (*c*), (*d*).

(2.) *To find the course.* Add together log. cos. mid. lat. and log. diff. long., and from the sum subtract log. true difference latitude; the remainder is the log. tan. course, which find in the tables, and mark it with the same letters as the true difference latitude and difference longitude. From the same opening take out the log. secant of course.

(3.) *To find distance.* To the log. secant course just found, add the log. true difference latitude; the sum (rejecting 10 in index) will be the log. distance.

EXAMPLES.

92. Required the course and distance from A to B, by middle latitude method.

lat. A 50° 25′ N.	long. A 27° 15′ W.
lat. B 47 12 N.	long. B 30 20 W.

lat. A . . 50° 25′ N.	. . . . . 50° 25′ N.	long. A 27° 15′ W.	
lat. B . . 47 12 N.	. . . . . 47 12 N.	long. B 30 20 W.	
3 13	2) 97 37	3 5	
60	mid. lat. 48 48	60	
T. D. lat. 193 S.		diff. long. 185 W.	

log. cos. mid. lat.	9·818681	log. sec. course	0·072849
log. diff. long. . .	2·267172	log. T. D. lat. .	2·285557
	12·085853	log. dist.	2·358406
log. T. D. lat. . .	2·285557	∴ dist. 228·2′	
log. tan. course .	9·800296	∴ course S. 32° 16′ W.	

Required the course and distance from A to B in each of the following examples, by middle latitude method:

	Lat. from and lat. in.	Long. from and long. in.	Answers. Course and dist.
93.	lat. A 49° 52′ S.	long. A 17° 22′ W.	N. 26° 40′ E.
	lat. B 42 13 S.	long. B 11 50 W.	513·6
94.	lat. A 21 15 S.	long. A 0 30 W.	S. 14° 37′ E.
	lat. B 30 27 S.	long. B 2 10 E.	570·5
95.	lat. A 60 15 S.	long. A 14 55 E.	S. 32° 50′ E.
	lat. B 65 36 S.	long. B 22 30 E.	382

Middle Latitude method.

Rule IV.

To find the latitude and longitude in, having given the course from a given place, and distance.

(1.) *To find latitude in.* Add together log. cos. course * and log. distance; the sum (rejecting 10 in the index) is

* Take out at the same time log. sin. course.

the log. true difference latitude, which find from tables, and mark N. or S. according as the course is northward or southward.

Apply true difference latitude (turned into degrees and minutes, if necessary) to the latitude from, and thus get latitude in. (Rule *e.*) Find the middle latitude. (Rule *c.*)

2. To find longitude in. Add together log. sin. course, log. distance, and log. secant middle latitude; the sum (rejecting 20 in the index) is the log. difference longitude, which find in tables, and mark E. or W. according as the course is eastward or westward. Apply the difference longitude (in degrees and minutes) to the longitude from, and thus get longitude in. (Rule *f.*)

EXAMPLE.

96. Sailed from A, S. 37° 10′ W., 472·6 miles; required the latitude in and longitude in (by middle lat. method).

lat. A	27° 20′ S.	long. A	25° 12′ W.
log. cos. course .	9·901394	log. sin. course. .	9·781134
log. dist.	2·674494	log. dist.	2·674494
log. T. D. lat. . .	2·575888	log. sec. mid. lat.	0·064531
∴ T. D. lat.	376·6′	log. diff. long. . .	2·520159
or	6° 17′ S.	∴ diff. long.	331·3′
lat. from	27 20 S.	or	5° 31′ W.
lat in . .	33 37 S.	long. from	25 12 W.
	2)60 57	long. in. .	30 43 W.
mid. lat.	30 28		

Required the latitude and longitude in, by middle latitude method, in each of the following examples, having sailed from A as follows:

	Course and dist. from A.	Lat. A.	Long. A.	Answers. Lat. in.	Long. in.
97.	N. 25°42′ W. 427·3′	64°10′ N.	40°15′ W.	70°35′ N.	48°17′ W.
98.	S. 48 58 W. 175·2	47 10 N.	32 15 W.	45 15 N.	35 26 W.
99.	N. 34 48 W. 383·7	50 25 N.	3 40 E.	55 40 N.	2 24 W.

Parallel Sailing.

In parallel sailing the ship is supposed to be kept on a parallel of latitude, as TS, fig. p. 3. The course will evidently be due east or due west, and the distance between two places as T and S, will be the arc TS between the two meridians passing through the places.

Rule V.

To find the course and distance, having given the latitude of the two places, and their longitudes.

(1.) Find the difference longitude.

(2.) The course is evidently due east or due west, according as the longitude in is to the east or west of longitude from.

(3.) *To find the distance.* Add together log. cos. latitude and log. difference longitude; the sum (rejecting 10 in index) is the log distance, which find in the table.

EXAMPLES.

100. Required the course and distance from A to B.

lat. A 80° N.	long. A 3° 50′ E.
lat. B 80 N.	long. B 6 10 W.

long. from	3° 50′ E.	dist. = diff. long. . cos. lat.	
long. in	6 10 W.	log. cos. lat. . . .	9·239670
	10 0	log. diff. long. . .	2·778151
	60	log. dist.	2·017821
	600 W.	∴ dist. 104·2′	

∴ the course is west.

Required the *compass* course and distance from A to B.

lat. A . . 50° 48′ N.	long. A . . 100° 0′ E.
lat. B . . 50 48 N.	long. B . . 101 0 E.

Variation of the compass two points E, and deviation as in table, p. 16.

long. A . .	100° E.	9·800737	
long. B . .	101 E.	1·778151	
	1	1·578888	
	60		
	60 E.		37·9 = distance.

	pts.	qrs.	
True course	8	0	r of N.
Variation	2	0	1
Compass course nearly	6	0	r of N = E.N.E.
Deviation	1	0	1
Compass course	5	0	r of N.

or N.E. by E.

Required the true course and distance from A to B, in each of the following examples:

	Lat. A and B.	Long. A.	Long. B.	Answers. Course and dist.
101.	70° 10′ S.	15° 10′ E.	22° 15′ E.	East 144·2′
102.	50 48 N.	5 0 W.	5 0 E.	East 379·2
103.	50 10 N.	40 25 W.	50 10 W.	West 374·7
104.	48 10 N.	100 0 W.	110 0 W.	West 400·2
105.	75 13 N.	15 20 E.	0 0 E.	West 234·7
106.	80 15 N.	179 0 E.	176 0 W.*	East 50·8

Parallel Sailing.

Rule VI.

To find the longitude in, having given the course and distance, and latitude and longitude from.

Add together log. sec. lat. and log. distance, the sum (rejecting 10 in the index) will be the log. difference longitude. Find the natural number thereof, and turn it into

* In this example it is evident we must modify the general rule; for the diff. long. is never considered to be greater than 180°. When, therefore, the above rule gives the diff. long. greater than 180°, subtract it from 360°, and apply thereto a contrary letter to the one directed by the rule; the result will be the diff. long. to be used.

degrees, and mark it E. or W. according as the course is E. or W. Apply difference longitude to longitude from, and thus find longitude in. (Rule *f.*)

The latitude in is the same as the latitude from.

EXAMPLE.

107. Sailed from A due east 1000 miles, required the latitude and longitude in.

lat. A 32° 10′ S. long. A. 28° 42′ W.

diff. long. = dist. . sec. lat.

log. sec. lat. . . . . . . 0·072372

log. dist. . . . . 3·000000

log. diff. long . . . . . . 3·072372

∴ diff. long. 1181′, or 19° 41′ E.

long. from . . . . 28 42 W.

∴ long. in . . . . . . 9 1 W.

and latitude in = lat. from = 32° 10′ S.

Required the latitude and longitude in, in each of the following examples:

	Course and dist.	Lat. from.	Long. from.	Answers. Lat. in.	Answers. Long. in.
108.	East 492·5′	52° 10′ N.	0° 29′ W.	52° 10′ N.	12° 54′ E.
109.	East 1752	60 0 N.	5 10 W.	60 0 N.	53 14 E.
110.	East 560	57 32 N.	13 5 W.	57 52 N.	4 18 W.
111.	West 740	60 0 N.	50 0 W.	60 0 N.	74 40 W.

The preceding rules are the principal ones used in navigation. It would be easy for the mathematical student to make for himself others, by means of the relations between the several terms course, dist., dep., &c., as shown by the fig. p. 151, in the author's volume of Astronomical Problems: he would find then no difficulty in solving problems similar to the following:

Sailed from A, in long. in 3° 10′ W., 300 miles due east, and altered my longitude 10 degrees; required the latitude and longitude in.

To find latitude.

$$\cos. \text{lat.} = \frac{\text{dist.}}{\text{diff. long.}} = \frac{300}{600} = \tfrac{1}{2}$$

$\therefore$ lat. in $= 60°$, and long. in $= 6° 50'$ E.

Wishing to make a small island, I took the ship to windward of it in the same latitude with the island, namely, 50° 48′ N. The longitude of the ship by chronometer was 20° 35′ W., and the long. of the island was 23° 50′ W. What was my distance from the island?

In this example of parallel sailing we have given lat. 50° 48′, and diff. long. 3° 15′, or 195′, to find distance.

dist. = diff. long. . cos. lat.

log. diff. long.	2·290035
log. cos. lat.	9·800737
log. dist.	2·090772

$\therefore$ dist. 123·2 miles.

The Day's Work.

To find the place of the ship at noon, that is, its latitude and longitude, having given the latitude and longitude at the preceding noon, the compass courses, and distances run in the interval, the deviation of the compass for each course on account of local attraction, the variation of the compass, the leeway, the velocity and direction of current (if any) &c., constitutes what is called the Day's Work.

Rule VII. (*the Day's Work*).

(1.) Correct each course for variation (Rule *g*), deviation (Rule *i*), and leeway (Rule *l*); thus get the true courses, and arrange the same in a tabular form, as in the example, p. 34. Add together the hourly distances sailed on each course, and insert the same in table opposite the true course.

(2.) Take out of the traverse table the true difference latitude and departure for each course and distance, putting

them down in the columns headed with the same letters as in course. Previously to opening the traverse table, fill up the columns of true difference latitude and departure not wanted, by drawing horizontal lines; this will frequently prevent mistakes.

(3.) If the ship does not sail from a place whose latitude and longitude are known, her bearing and distance from some near object, as a church-spire, &c., must be ascertained, and also its latitude and longitude. Then the ship is supposed to sail from this known object to her anchorage, her course being the opposite to the bearing of the object from the ship. This course must be corrected like the rest for variation and deviation, and inserted in the table as an actual course, with the distance of the object as a distance.

(4.) If a current sets the ship in any ascertained direction, and with a known velocity, these also may be conceived to be an independent course and distance, and must be corrected for variation, and should be for deviation also, if the latter correction is appreciable, which is rarely the case.

(5.) *To find the latitude in.* The quantities in the four columns of true difference latitude and departure being added up separately, the difference between the north difference of latitude and south difference of latitude, with the name of the greater, will give the true difference of latitude, made at the end of the day. The departure is found in a similar manner. Apply true difference latitude to latitude from, so as to obtain the latitude in.

(6.) *To find the longitude in.* Add together log. sec. mid. lat. and log. departure, the result (rejecting 10 in the index) is the log. difference longitude. Find this in table, and thus the longitude in is found.*

The following example, worked out in detail, will perhaps

* Or thus:—To find diff. long., add together log. M.D. lat. and log. dep., and from the sum subtract log. T. D. lat.; the remainder is the log. diff. long., which find in the tables.

be sufficient to explain the operations directed in the above general rule.

EXAMPLE.

112. April 27th, 1852, at noon. A point of land in latitude 36° 30′ S. and longitude 110° 20′ W. bore by compass E.N.E.½E. (ship's head being S.E. by S. by compass), distant 14 miles; afterwards sailed as by the following log account; required the latitude and longitude in, on April 28th, at noon.

Hours.	Knots.	Courses.	Winds.	Lee-way.	Devi-ation.	Remarks.
1	2·5	S.W.½W.	S.b.E.	2¼	½ l.	P.M.
2	3·4					
3	2·3					
4	3·2	W.b.S.½S.	S.b.W.	2½	¾ l.	
5	4·4					
6	2·3					Variation of com-
7	2·3	W.b.N.¾N.	S.W.	2	¾ l.	pass 1¾ E.
8	3·3					
9	4·0					
10	5·4					
11	4·2					
12	4·4					
1	3·3	N.W.½W.	W.b.S.¾S.	2½	½ l.	A.M.
2	3·3					
3	3·5					
4	4·2					A current set the
5	6·3	W.b.S.	S.½W.	1½	¾ l.	ship the last 8
6	3·7					hours, by com-
7	2·5					pass, E.½S., 2
8	5·0					miles an hour.
9	5·2	S.W.	S.b.E.	2½	½ l.	
10	3·4					
11	6·3					
12	5·4					

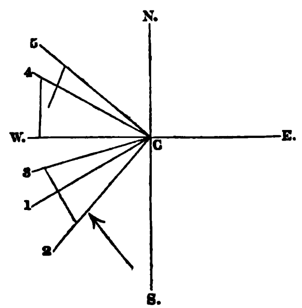
(1.) The column in the above table headed deviation should be formed from the general table of deviations (p. 16) previously to correcting courses. Thus, in the first course in the preceding table, the ship's head is S.W.½W.; looking in the deviation table we see that the corresponding correction is ½ W. or ½ l. (Rule *i.*)

(2.) Form a table such as below, by writing down the headings, points, courses, &c., over the seven columns which are to be filled in with the corrected courses, &c.

Points.	Courses.	Distance.	Diff. lat.		Departure.	
			N.	S.	E.	W.

(3.) *To correct the courses.*

The courses are more readily corrected by drawing two lines at right angles, to represent the N., S., E., and W. points of the compass, and then a line to represent (roughly) the compass course of the ship. The direction in which the correction for leeway must be applied will then be easily seen.



After some experience in correcting courses, they can be made mentally, and the diagram dispensed with.

To correct the departure course which is W.S.W.½W. (the opposite bearing to E.N.E.½E.).

Draw a line roughly in the fig. W.S.W.½W. as C1; it is then seen that

			pts.	qrs.
compass course			6	2 r. S.
variation	1	3 r.		
ship's head S.E.b.S.. ∴ dev.	0	0		(See table, p. 16.)
			1	3 r.
true course			8	1 r. S.

or 7 pts. 3 qrs. left of N., or W.¼N. dist. 14′.

Insert this course and distance in table below.

Points.	Courses.	Dist.	Diff. lat.		Departure.	
			N.	S.	E.	W.
7¾	W.¼N.	14·0	0·7	...	...	14·0
8	W.	8·2	...	...	...	8·2
6	W.N.W.	9·9	3·8	...	...	9·2
3¼	N.W.b.N.¼W.	23·6	19·0	...	...	14·1
¾	N.¾W.	14·3	14·1	...	...	2·1
6½	W.N.W.½W.	17·5	5·1	...	...	16·7
7¾	W.¼S.	20·3	...	1·0	...	20·3
5¾	S.E.b.E.¾E.	16·0	...	6·8	14·5	...
			42·7	7·8	14·5	84·6
			7·8			14·5
		T.D. lat. 34·9 N.			Dep. 70·1 W.	

First Course.—S.W.½W.

Draw a line in fig. S.W.½W. as C2; then

			pts.	qrs.
compass course			4	2 r. S.
variation	1	3 r.		
deviation	0	2 l.		
			1	1 r.
			5	3 r. S.
leeway (wind S.b.E.)			2	1 r.
true course			8	0 r. S. or due W. 8·2′.

The distance 8·2′ is found by adding up the hourly distances until the course is altered, at 4 o'clock. Insert this course and distance in the table.

Second Course.—W.b.S.½S.

Draw a line in fig. W.b.S.½S. as C3.

			pts.	qrs.	
compass course			6	2	r. S.
variation	1	3 r.			
deviation	0	3 l.			
			1	0	r.
			7	2	r. S.
leeway			2	2	r.
true course			10	0	r. S.
		or	6	0	l. N.
			= W.N.W. 9·9′.		

Insert this course and distance in the table.

Third Course.—W.b.N.¾N.

Draw a line W.b.N.¾N. as C4.

			pts.	qrs.	
compass course			6	1	l. N.
variation	1	3 r.			
deviation	0	3 l.			
			1	0	r.
			5	1	l. N.
leeway			2	0	r.
true course			3	1	l. N.
			or N.W.b.N.¼W. 23·6′.		

Insert this course and distance in table.

Fourth Course.—N.W.½W.

Draw a line N.W.½W. as C5.

			pts.	qrs.	
compass course			4	2	l. N.
variation	1	3 r.			
deviation . . .	0	2 l.			
			1	1	r.
			3	1	l. N.
leeway			2	2	r.
true course			0	3	l. N.
			or N.¾W. 14·3′.		

Insert this course and distance in the table.

Proceed with the 5th and 6th courses in the same manner, thus:

Fifth Course.				*Sixth Course.*			
	pts.	qrs.			pts.	qrs.	
W.b.S.	7	0	r. S. as C3.	S.W. .	4	0	r. S.
1 3 r.				1 3 r.			
0 3 l.				0 2 l.			
——	1	0	r.	——	1	1	r.
	8	0	r. S.		5	1	r. S.
	1	2	r.		2	2	r.
	9	2	r. S.		7	3	r. S.
or	6	2	l. N.				or W.¼S. 20·3′.
= W.N.W.½W. 17·5′.							

	pts.	qrs.	
Current course E.½S.	7	2	l. S.
variation	1	3	r.
true course	5	3	l. S.

or S.E.b.E.¾E. 16·0′.

Previously to opening the traverse table to take out the difference latitude and departure corresponding to each course and distance in the above table, fill up the columns not wanted: thus in the first course W.¼N. the N. and W. columns will be wanted; fill up the S. and E. columns by drawing a line under S. and E. In the second course W. the three columns N., S., and E., will not be wanted; fill them up with lines. In the same manner proceed with the other courses.

(4.) *To find difference latitude and departure* for each course and distance, by traverse table.

Enter traverse table, and take out the difference latitude and departure corresponding to 7¾ points, and distance 14·0.

(Look out rather $7\frac{3}{4}$ points and 140 distance, the diff. lat. and dep. for which are 6·9 and 139·8; move the decimal points one place to the left,) and put down the result to the nearest tenth, which are ·7 and 14·0. Insert them in the spaces left unmarked under N. and W.

The second course being due W. 8·2′, the departure will be 8·2 (the same as the distance).

With third course 6 points and distance 9·9 (looking for 99, and making the proper change in decimal points) the diff. lat. is 3·8′ and dep. 9·2′.

In a similar manner find difference latitude and departure for the other courses.

When the four columns are added up, it appears that the ship has sailed N. 42·7′ and S. 7·8′; therefore upon the whole the true difference latitude is 34·9′ N.; and her departure has been 14·5′ E. and 84·6′ W.; hence the departure made good in the 24 hours is 70·1′ W.

(5.) *To find the latitude in*, apply the true difference latitude to the latitude from, in the usual manner, to obtain the latitude in. (Rule *e*.)

(6.) *To find the longitude in.** With the latitude from and latitude in, find middle latitude. Add together log. secant mid. lat. and log. departure; the result (rejecting 10 in index) is the log. difference longitude, which, found in

* Or thus: To find long. in (by inspection).

$$\text{Since } \frac{\text{dep.}}{\text{dist.}} = \text{sin. course}$$

$$\text{and } \frac{\text{dep.}}{\text{diff. long.}} = \text{cos. mid. lat.} = \text{sin. complement mid. lat.}$$

If, therefore, the traverse table is entered with complement of mid. lat. as a course, and with the given departure, the distance corresponding thereto will be the difference of longitude nearly.

the tables, and applied to the longitude from, gives the longitude in. Thus:

To find latitude in.		To find longitude in.	
T. D. lat. .	0° 34′ 54″ N.	log. sec. mid. lat.	0·093148
lat. from .	36 30 0 S.	log. departure . .	1·845718
lat. in. . .	35 55 6 S.	log. diff. long. . .	1·938866
	2)72 25 6	∴ diff. long.	87′
mid. lat. .	36 12 33	or	1° 27′ W.
		long. from . .	110 20 W.
		long. in. . . .	111 47 W.

Mercator's Chart.

To find course and distance on Mercator's chart between two known places.

To find course. Apply the edge of a ruler to the two places, and then ascertain at what degree a straight line (as the edge of the same or another ruler) placed parallel thereto, and passing through the centre of some adjacent compass, cuts the circumference. This will indicate the bearing or course required.

To find distance. This may, in general, be found by applying the distance on the chart to the side of chart, so that the chart distance may be so placed that the middle point may coincide with the middle parallel between the two places; then the degrees of lat. covered by the chart distance will be the distance nearly.

More concise methods for solving many of the preceding problems might have been given, by employing the traverse table, &c. But these will suggest themselves to the learner after he has had some practical experience in nautical matters.

The following examples formed part of examination papers given at the Royal Naval College, at the monthly examination, in navigation, of lieutenants and assistant-masters in her Majesty's Royal Navy.

113. Required the course and distance from A to B.

lat. A . . . 50° 25′ N.		long. A . . . 3° 40′ E.	
lat. B . . . 55 40 N.		long. B . . . 2 25 W.	

114. Find the course (by compass) and dist. from A to B.

lat. A . . . 70° 10′ N.	long. A . . . 15° 5′ E.
lat. B . . . 70 10 N.	long. B . . . 20 5 E.

The deviation is given in table, p. 16.

115. June 20, 1852, at noon, a point of land in latitude 47° 12′ N. and longitude by account 3° 10′ W. bore by compass E.N.E., the ship's head being South by compass, distant 17 miles (variation of the compass 3¼ W.); afterwards sailed as by the following log account: required the lat. and long. in, on June 21, at noon.

Hours.	Knots.	$\frac{1}{10}$ths.	Course.	Wind.	Lee-way.	
1	3	5	E.b.N.¼N.	N.½E.	2	
2	3	2				
3	4	0				
4	3	6				Variation of the compass 3¼ W. For local deviation see table, p. 16.
5	3	5	W.N.W.	ditto.	1¾	
6	4	1				
7	3	6				
8	3	7				
9	4	2	S.S.W.	W.	2¼	
10	4	1				
11	3	6				
12	3	2				
1	3	6	N.N.W.	W.	1¾	
2	3	2				A current set the ship the last 4 hours, by comp. S. S. W. ½ W., 2 miles an hour. The current need not be corrected for local deviation.
3	4	0				
4	4	3				
5	4	2	S.E.½E.	S.S.W.	1½	
6	3	6				
7	3	6				
8	4	2				
9	3	5	S.W.¾W.	S.b.E.	3¼	
10	4	2				
11	4	0				
12	5	2				

116. Required the course and distance from A to B.

lat. A . . . 47° 50′ N. long. A . . . 32° 20′ W.
lat. B . . . 45 10 N. long. B . . . 35 40 W.

117. Required the course (by comp.) and dist. from A to B.

lat. A . . . 70° 10′ S. long. A . . . 5° 0′ W.
lat. B . . . 70 10 S. long. B . . . 5 0 E.

Variation of compass 2½ W. For local dev. see p. 16.

118. March 5, 1852, at noon, a point of land in latitude 57° 12′ N. and longitude by account 75° 34′ W. bore by compass E.N.E. (ship's head being N. by compass), distant 18 miles (variation of the compass 1½ W.); afterwards sailed as by the following log account; required the lat. and long. in, on March 6, at noon.

Hours.	Knots.	$\frac{1}{10}$ths.	Course.	Wind.	Lee-way.	
1	5	0	N. ½W.	W. b. N.	1¾	
2	4	5				
3	4	2				
4	3	7				Variation of the
5	2	0	S. W. b. S.	ditto.	2¼	compass 1½ W.
6	2	3				For local devia-
7	2	4				tion see table,
8	3	0	N. N. E. ½E.	N. W. ½N.	2	p. 16.
9	3	2				
10	2	5				
11	3	2				
12	2	6				
1	2	0	W. ½N.	N. N. W.	2¾	
2	2	6				A current set the
3	2	1				ship the last 6
4	2	7				hours, 4 miles an
5	5	0	S. E. ½E.	S. W.	0	hour. N. N. W.
6	6	2				(by compass.)
7	6	3				The current need
8	7	0				not be corrected
9	1	0	S. ½W.	W. S. W.	3	for local devia-
10	1	2				tion.
11	1	5				
12	1	6				

119. Required the course and distance from A to B.

lat. A . . . 72° 20′ S. long. A . . . 13° 25′ W.
lat. B . . . 65 42 S. long. B . . . 20 10 W.

120. Find the course (by compass) and dist. from A to B.

lat. A . . . 70° 10′ N. long. A . . . 15° 5′ E.
lat. B . . . 70 10 N. long. B . . . 20 5 E.

Variation of compass 2 W. For local deviation, see p. 16.

121. June 1, 1852, in longitude 18° 28′ E., and latitude 34° 28′ S., a point of land bore N.W. (ship's head N. by compass), distant 10 miles (variation of the compass 2¼ W.); afterwards sailed as per log: required the latitude and longitude in, on June 2.

Hours.	Knots.	$\frac{1}{10}$ths.	Course.	Wind.	Lee-way.	
1	5	4	N.b.E.½E.	N.W.½W.	2¼	
2	5	2				
3	5	8				
4	6	1				Variation of the compass 2¼ W. For local deviation see table, p. 16.
5	6	5	S.S.W.	N.W.	¼	
6	7	3				
7	7	0				
8	7	2				
9	6	8	N.W.b.W.	S.E.	0	
10	6	5				
11	6	1				
12	5	8	S.b.W.¾W.	S.E.½E.	2¼	
1	6	0				
2	6	5				A current set the ship the last 5 hours, by compass N.W., 2 miles an hour.
3	6	8				
4	6	4	N.N.E.	N.W.	2	
5	6	0				
6	6	5				
7	6	8	N.W.	E.	0	The current need not be corrected for local deviation.
8	2	4				
9	3	6				
10	4	7				
11	3	5				
12	2	2				

122. Required the course and distance from A to B.

lat. A . . . 3° 30′ N. long. A . . . 74° 40′ E.
lat. B . . . 2 20 S. long. B . . . 59 17 E.

123. A ship sailed from A 380 miles E.S.E.½E.: required the latitude and longitude in.

lat. A . . 39° 12′ N. long. A . . . 78° 50′ W.

124. May 2, 1852, at noon, a point of land, in latitude 55° 10′ S. and longitude 67° 20′ W., bore by compass N.E.¾E., ship's head N., distant 12 miles (variation of the compass 3¾ W.); afterwards sailed as by the following log account: required the latitude and longitude in, on May 3.

Hours.	Knots.	$\frac{1}{10}$ths.	Course.	Wind.	Lee-way.	
1	3	8	N.	W.N.W.	3½	
2	3	7				
3	4	1				
4	3	9	W.S.W.¾W.	N.W.b.N.	2¾	Variation of the
5	3	8				compass 3¾ W.
6	4	0				For local devia-
7	4	1				tion see table,
8	3	7	N.W.½W.	N.N.E.	2½	p. 16.
9	2	9				
10	2	8				
11	2	5				
12	1	7	E.	S.b.E.½E.	2½	
1	1	7				
2	1	6				A current set the
3	2	4				ship the last 5
4	2	5	S.S.W.¾W.	S.E.½S.	3	hours, by com-
5	2	7				pass 4½ miles an
6	3	9				hour, E.½S.
7	3	8				The current need
8	2	7				not be corrected
9	3	4	N.¾E.	E.N.E.	4	for local devia-
10	4	3				tion.
11	4	3				
12	4	6				

125. Required the course and distance from A to B.

lat. A . . . 21° 15′ S. long. A . . . 0° 30′ W.
lat. B . . . 30 27 S. long. B . . . 2 10 E.

126. Required the compass course and dist. from A to B.

lat. A . . . 15° 30′ N. long. A . . . 2° 10′ E.
lat. B . . . 15 30 N. long. B . . . 3 40 E.

Variation of compass $2\frac{1}{2}$ E. For local deviation see p. 16.

127. March 6, 1852, at noon, a point of land, in latitude 47° 10′ N. and longitude by account 10° 46′ W., bore by compass E.b.N.$\frac{1}{2}$N. (ship's head being S.S.E. by compass), distant 20 miles (variation of the compass $2\frac{1}{4}$ W.); afterwards sailed as by the following log account: required the lat. and long. in on March 7, at noon.

Hours.	Knots.	$\frac{1}{10}$ths.	Course.	Wind.	Lee-way.	
1	3	2	S.S.W.$\frac{1}{4}$W.	S. E.	2	
2	3	5				
3	4	0				
4	4	0	S.W.	S. E. b. S.	$1\frac{1}{4}$	Variation of the compass $2\frac{1}{4}$ W.
5	3	6				For local deviation see table, p. 16.
6	3	7				
7	3	2	E. N.E.	S. E.	$2\frac{1}{4}$	
8	3	0				
9	4	0				
10	4	2				
11	7	0	N.E.	S.E.	0	
12	8	2				
1	7	2	N.E.$\frac{1}{4}$E.	N.N.W.	2	
2	3	4				A current set the ship the last 4 hours, by compass W. b. N., 2 miles an hour.
3	4	2				
4	3	7				
5	4	0	N.E.b.E.	N.b.W.	$2\frac{1}{4}$	
6	3	6				
7	3	7				The current need not be corrected for local deviation.
8	4	2				
9	4	0				
10	8	0	N.W.	S.	0	
11	9	–				
12	10	–				

128. Required the course and distance from A to B.

lat. A . . . 9° 30′ S. long. A . . . 2° 4′ W.
lat. B . . . 7 10 S. long. B . . . 1 30 E.

129. Two places in the same latitude N. whose difference of longitude is 700 miles, are distant from each other 400 miles; required the latitude they are in. (See p. 30.)

130. Jan. 10, 1852, at noon, a point of land, in latitude 46° 12′ S. and longitude by account 2° 10′ W., bore by compass E.b.S.$\frac{1}{2}$S., distant 20 miles, the ship's head being East by compass; afterwards sailed as by the following log account: required the latitude and longitude in, on Jan. 11, at noon.

Hours.	Knots.	$\frac{1}{10}$ths.	Course.	Wind.	Lee-way.	
1	3	5	S.W.$\frac{1}{2}$W.	S.b.E.$\frac{1}{2}$E.	2	Variation of the compass 1$\frac{1}{4}$ E. For local deviation see table, p. 16.
2	3	2				
3	3	6				
4	3	7				
5	3	0	N.$\frac{3}{4}$E.	E.N.E.	$\frac{1}{4}$	
6	3	2				
7	3	6				
8	4	–				
9	3	2	S.b.E.$\frac{1}{2}$E.	S.W.$\frac{1}{2}$W.	2$\frac{3}{4}$	
10	2	5				
11	2	1				
12	1	6				
1	3	2	W.b.S.	S.b.W.	2$\frac{1}{4}$	A current set the ship the last 5 hours, by compass, S.W.$\frac{1}{4}$W., 4$\frac{1}{2}$ miles an hour. The current need not be corrected for local deviation.
2	3	–				
3	2	6				
4	3	5				
5	3	2	E.N.E.	S.E.	2$\frac{1}{2}$	
6	3	5				
7	4	0				
8	3	6	S.S.W.$\frac{1}{2}$W.	S.E.	1$\frac{3}{4}$	
9	3	7				
10	2	2				
11	2	6				
12	2	7				

In the following examination paper the ship's compasses are not supposed to be affected by the iron on board.

131. Required the course and distance from A to B.

lat. A . . . 78° 12′ N.   long. A . . . 80° 15′ W.
lat. B . . . 73 50 N.   long. B . . . 89 10 W.

132. Sailed from A 555·6 miles due N.; required the latitude and longitude in.

lat. A . . . 3° 15′ S.   long. A . . . 100° W.

133. On June 5, 1852, at noon, a point of land, in latitude 70° 15′ N. and longitude 7° 13′ E., bore by compass N.W.¼W., distant 18 miles (variation 1½ W.); afterwards sailed as by the following log account: required the latitude and longitude in, on June 6, at noon.

Hours.	Knots.	$\frac{1}{10}$ths.	Course.	Wind.	Lee-way.	
1	3	5	N.¼W.	W.b.N.¾N.	2¼	P.M.
2	3	2				
3	2	7				
4	3	–				
5	4	–	N.N.E.	E.	2½	
6	2	5				
7	3	–				Variation 1½ W.
8	2	6				
9	3	–	S.S.W.½W.	S.E.	1½	
10	2	5				
11	3	–				
12	2	–				
1	8	–	S.½W.	N.	0	A.M.
2	8	–				
3	8	–				
4	8	–				
5	8	–				
6	8	–				
7	8	–				
8	7	5				
9	8	–				
10	7	–				
11	8	–				
12	9	–				

ANSWERS TO QUESTIONS 113—133.

113. Course N. 34° 49′ W., distance 384·5.

114. Compass course E.¾N., distance 101·8.

115. True courses, &c., S.S.W. 17·0′; E.N.E.¼E. 14·3′; S.W.½W. 14·9′; S.E. 15·1′; N.W.½N. 15·1′; E.b.N. 15·6′; S.W.¼W. 16·9′; S.¾E. 8·0′. Lat. in 46° 37′ N., long. in 3° 9′ W.

116. Course S. 40° 42′ W., dist. 211·0.

117. Compass course S.E.b.E.¾E., dist. 203·6.

118. True courses, &c., S. W. ¾ W. 18·0′: North 17·4′; S. b. E.¼E. 6·7′; N. E.¼N. 14·5′; S.W.½S. 9·4′; E.S.E.¼S. 24·5′; S. b. E.¼E. 5·3′; N.W.½W. 24·0′. Lat. in 57° 20′ N., long. in 75° 31′ W.

119. Course N. 19° 52′ W., distance 423·2.

120. Compass course E.S.E.½E., distance 101·8.

121. True courses, &c., E.S.E. 10·0′; N.N.E. 22·5′; S.b.E. 28·0′; W.¼N. 19·4′; S.b.W.¼W. 25·1′; N.N.E.½E. 18·9′; W. N. W. ¾ W. 23·2′; W. N. W. ¼ W. 10·0′. Lat. in 34° 36′ S., long. in 17° 57′ E.

122. S. 69° 14′ W., 987·1.

123. Lat. in 37° 22′ N., long. in 71° 8′ W.

124. True courses, &c., S. b. W. ¼ W. 12·0′; North 11·6′; S. ½ E. 15·8′; S.W.¾W. 11·9′; N. N. E. ½ E. 7·4′; S. b. W. ½W. 15·6′; W.N.W.½W. 16·6′; N. E. ¾ E. 22·5′. Lat. in 55° 23′ S., long. in 67° 37′ W..

125. S. 14° 36′ E., 570·4.

126. N.E.½E. 86·7.

127. True courses, &c., S.W.¼W. 20·0′; S.b.W.¾W. 10·7′; S. S.W.½W. 11·3′; N.N.E.¼E. 14·4′; N.N.E.½E. 15·2′; N.E.¾E. 18·5′; E.N.E. 19·5′; W.N.W.¾W. 27·0′; W.S.W.¾W. 8·0′. Lat. in 47° 26′ N., long. in 11° 3′ W.

128. N. 56° 32′ E., 253·9.

129. Lat. 55° 9′ N. (See similar Ex. p. 30.) This example is worked out as follows: From the logarithm of the distance (increased by 10) subtract log. diff. long.: the remainder is the log. cos. latitude, which find in the tables.

130. True courses, &c., N.W.½W. 20·0′; W. b. S.¼S. 14·0′; N.N.E.¼E. 13·8′; S.E.b.S.¼S. 9·4′; W.N.W.¼W. 12·3′; E. N. E. ¼ N. 10·7′; S. W. b. W. 14·8′; S.W. b.W. ½W. 22·5′. Lat. in 46° 6′ S.; long. in 3° 24′ W.

131. S. 26° 4′ W., 291·7.

132. Lat. 6° 0′ 36″ N.; long. 100° W.

133. True courses, &c., S.E. b. E.¾ E. 18′ dep. course; N.½E. 12·4′; N. N.W. 12·1′; S. S.W.½W. 10·5′; S. b. E. 95·5′. Lat. in 68° 48′ N.; long. in 8° 29′ E.

NAUTICAL ASTRONOMY.

NAUTICAL ASTRONOMY.

SECTION I.

DEFINITIONS IN NAUTICAL ASTRONOMY, EXPLANATION AND USE OF NAUTICAL ALMANAC, AND PRELIMINARY PROBLEMS.

CHAPTER I.

ASTRONOMICAL AND NAUTICAL TERMS AND DEFINITIONS.

1. NAUTICAL ASTRONOMY teaches the method of finding the place of a ship by means of astronomical observations.

2. The following pages will contain the principal rules for determining the latitude and longitude and the variation of the compass, and a series of examples under each rule will be given for practice. The requisite elements from the Nautical Almanac will be printed after each batch of examples, in order that the student may learn as early as possible the use of that important volume.

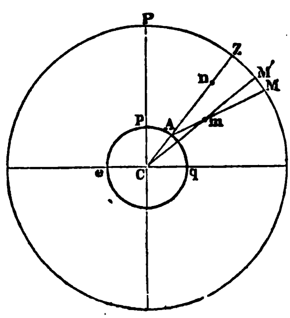
3. By the combination of theory with astronomical observations, the motions of the sun, moon, and planets have been determined with great accuracy, so that their places may be computed beforehand. An account of these motions and relative positions of the heavenly bodies is printed every year in England, under the name of the *Nautical Almanac*. In France a similar work is published, called the *Connaissance des Tems*.

Before we enter upon the explanation of the contents and uses of the Nautical Almanac, we will give definitions of the most important terms used in Nautical Astronomy.

Astronomical Terms and Definitions.

4. To a spectator on the earth the sun, moon, and stars seem to be placed on the interior surface of a hollow sphere of great but indefinite magnitude. The interior surface of this sphere is called the *celestial concave*, the centre of which may be supposed to be the same as that of the earth.

5. The heavenly bodies are not in reality thus situated with respect to the spectator; for they are interspersed in infinite space at very different distances from him: the whole is an optical deception, by which an observer, wherever he is placed, is induced to imagine himself to be the centre of the universe. For let us suppose that *e p q* be the earth, P Z M the celestial concave, and *m* and *n* heavenly bodies at different distances from a spectator placed at A. Then the spectator not being able to estimate the relative distances of *m* and *n*, would imagine both the bodies to be situated in the celestial concave at Z and M, at the same distance from him. This figure will enable us to explain the terms *true* and *apparent place* of a heavenly body. The body *m* viewed from the surface of the earth would appear to a spectator A to be at M in the celestial concave: if it were seen from the centre of the earth, the point occupied by *m* would be M′, the extremity of a line drawn from the centre C of the earth through the heavenly body to the celestial concave. M is called the *apparent* place, and M′ the *true* place of the heavenly body *m*.



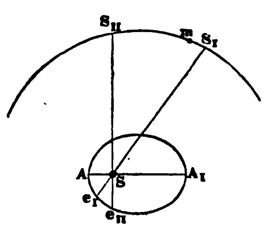
6. The *axis of the earth* is that diameter about which it revolves: the poles of the earth are the extremities of the axis.

7. The *terrestrial equator* is that great circle on the earth that is equidistant from each pole.

8. A spectator on the earth, not being sensible of the motion by which in fact he describes daily a circle from west to east with the spot on which he stands, views in appearance the heavens moving past him in the opposite direction, or from east to west. The sphere of the fixed stars, or as it is more usually called, the celestial concave, thus appears to revolve from east to west round a line which is the axis of the earth produced to the celestial concave: this line is therefore called the *axis of the heavens*.

9. *The poles of the heavens* are the extremities of the axis of the heavens.

10. *The celestial equator* is that great circle in the celestial concave which is perpendicular to the axis of the heavens; or it may be defined to be the terrestrial equator expanded or extended to the celestial concave. The poles of the celestial equator and the poles of the heavens are therefore identical. While the earth thus performs its daily revolution, it is carried with great velocity from west to east round the sun, and describes an elliptic orbit once every year. This annual motion of the earth round the sun, causes the latter body, to a spectator on the earth, insensible of his own change of place, to appear to describe a great circle in the celestial concave from west to east. This may be explained by a figure. Let A $e_{,}$ $A_{,}$ be the earth's orbit, S the sun, and $S_{,}$ m $S_{,,}$ the celestial concave; then, to a spectator at $e_{,}$, the sun is seen at a point $S_{,}$ in the celestial concave, a little, we will suppose, to the west of a fixed star at m; but when

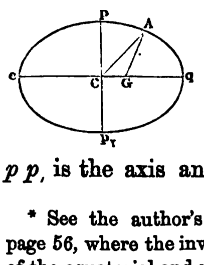


the earth has arrived at $e_{\prime\prime}$, the spectator (not being sensible of his motion from $e_{\prime}$ to $e_{\prime\prime}$) imagines the sun to be at $s_{\prime\prime}$, to the east of the star *m*, and to have described the arc $s_{\prime}\ s_{\prime\prime}$ in the time the earth moved from $e_{\prime}$ to $e_{\prime\prime}$. It appears from this, that when the earth has arrived again at $e_{\prime}$ the sun will again be at $s_{\prime}$, having described one complete circle in the celestial concave among the fixed stars. The great circle thus described by the sun is called the *ecliptic.*

11. The axis of the earth as it is thus carried round the sun, continues always parallel to itself, and is supposed, on account of the smallness of the earth's orbit (small, when compared with the distance of the heavenly bodies), to be always directed to the same points in the celestial concave, namely, the *poles* of the heavens.

12. From observation, the celestial equator is found to be inclined to the ecliptic at an angle of about 23° 28′. This inclination of the equator to the ecliptic is called the *obliquity of the ecliptic.* The axis of the earth, therefore, which is perpendicular to the equator, is inclined to the ecliptic, or, as it is in the same plane, to the earth's orbit, at an angle of 66° 32′.

13. In consequence of the whirling motion of the earth about its axis, the parts near the equator, which have the greatest velocity, acquire thereby a greater distance from the centre than the parts near the poles. By actual measurement of a degree of latitude in different parts of the earth, it is found that the equatorial diameter is longer than the axis or polar diameter by 26 miles: the former being about 7924 miles; the latter about 7898 miles,* and that the form of the earth is that of an *oblate spheroid* resembling the annexed figure, in which



$p\ p_{\prime}$ is the axis and $e\ q$ the equator. It is usual, however,

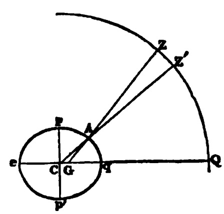
* See the author's Problems in Astronomy, &c., and Solutions, page 56, where the investigation of this problem is given, and the values of the equatorial and polar diameters calculated.

in drawing the figure of the earth to exaggerate very much its ellipticity; this is done for the sake of drawing the lines about the figure with greater clearness; for if it were constructed according to its true dimensions, the line $p\,p_{,}$ (being only about the $\frac{1}{300}$th part of itself less than $e\,q$) would appear to the eye of the same length as $e\,q$, and we should see that the figure that more nearly resembles the earth would be a sphere.

14. If a perpendicular A G be drawn to the earth's surface passing through A, the angle A G q, formed by the line with the plane of the equator is *the latitude* of the point A.

If a line be drawn from A to C, the centre of the earth, the angle A C q is called the *reduced*, or central latitude of A. The difference between the true and reduced latitude is not great: it is, however, of importance in some of the problems in Nautical Astronomy. This correction has accordingly been calculated,* and forms one of the Nautical Tables.

Sections of the earth passing through the poles, as p A q, are called meridians of the earth. If the earth is considered as a sphere (which it is very nearly), the meridians will be circles: on this supposition, moreover, the perpendicular A G would coincide with A C, and the latitude of a place on the surface of the earth may be, on this supposition, defined to be the arc of the meridian passing through the place, intercepted between the place and the equator. If G A be produced to meet the celestial concave at Z, the point Z is the zenith of the spectator at A. If C A be produced to the celestial concave at Z′, then Z′ is called the *reduced* zenith of the spectator at A. The point opposite to Z in the celestial concave is called the *Nadir*. In the figure the terrestrial



* See the author's Astronomical Problems and Solutions, page 59, for the investigation of this correction.

equator *e q* is extended to the celestial concave, and therefore *e* C Q is the plane of the *celestial* equator.

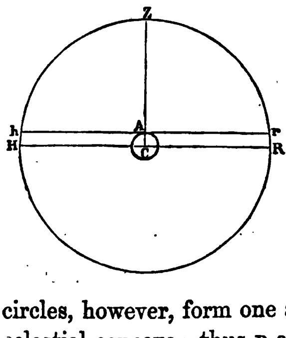
By means of the figure we may define the zenith, reduced zenith, latitude, and reduced latitude, as follows:—

The *zenith* is that point in the celestial equator which is the extremity of the line drawn perpendicular to the place of the spectator, as Z.

The *reduced* zenith is that point in the celestial concavewhich is the extremity of a straight line drawn from the centre of the earth, through the place of the spectator, as Z′.

The *latitude* of a place A on the surface of the earth, is the inclination of the perpendicular A G to the plane of the equator: thus the angle A G Q is the latitude of A. The arc Z Q in the celestial concave measures the angle A G Q; hence Z Q, *or the distance of the zenith from the celestial equator, is equal to the latitude of the spectator.*

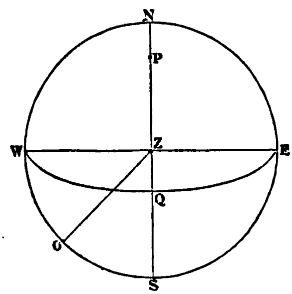
The *reduced latitude* of the place A, is the inclination of Z′ C or A C to the plane of the equator: or it is the angle A C Q or arc Z′ Q, which measures the angle. Since the curvature of the earth diminishes from the equator to the poles, the reduced latitude Z′ Q must be always less than the true latitude Z Q, and therefore the difference Z Z′ must be subtracted from the true latitude to get the reduced latitude.



The *visible horizon* is that circle in the celestial concave which touches the earth where the spectator stands, as *h* A *r*; and a circle parallel to the visible horizon, and passing through the centre of the earth, is called the rational horizon: thus H C R is the rational horizon. These two circles, however, form one and the same great circle in the celestial concave: thus R and *r* in the figure must be sup-

posed to coincide. This may be readily conceived, when we consider that the distance of any two points on the surface of the earth will make no sensible angle at the celestial concave; therefore either of these two circles is to be understood by the word horizon. The *poles* of the horizon of any place are manifestly the zenith and nadir.

Great circles passing through the zenith are called *circles of altitude* or *vertical* circles. Thus, let Z be the zenith of a spectator, where the horizon is represented by the circle N W S E, then N Z S, W Z E, and Z O are circles of altitude. That circle of altitude which passes through the poles of the heavens is called the *celestial meridian*. Thus, suppose the point P to be that pole of the heavens which is above the horizon (and therefore called the elevated pole), then the circle N P Z S is the celestial meridian of a spectator supposed to be on the earth below Z.

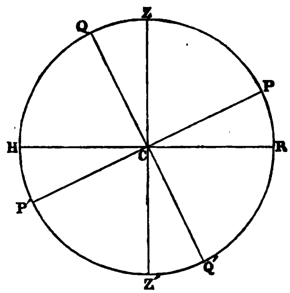


The points of the horizon through which the celestial meridian passes are called the *north* and *south* points. A circle of altitude at right angles to the meridian is called the *prime vertical*: thus W Z E is the prime vertical. This last circle cuts the horizon in two points called the *east* and *west* points. The east and west points are manifestly the poles of the celestial meridian.

Since the horizon and celestial equator are both perpendicular to the celestial meridian, the points where the horizon and celestial equator intersect each other, must be 90° distant from every part of the meridian (Jeans' Trig., P. II, art. 65); that is, the celestial equator must cut the horizon in the east and west points. If, therefore, P is the pole of the heavens, take P Q = 90°, then the celestial equator must pass through Q, and as we see it must also pass through

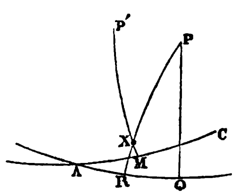
the east and west points, the curve W Q E, in the figure, will represent the celestial equator.

The preceding terms are frequently explained by means of a figure projected on the plane of the celestial meridian, as thus: Let the circle P Z Q represent the celestial meridian, the pole C is either the east or west point; let H R be the horizon and P the pole of the heavens above the horizon; the line P P′ may represent the axis of the heavens, and Q Q′, drawn at right angles to it, the celestial equator; the poles of the horizon will be Z and Z′, the zenith and the nadir, and Z C Z′, the circle passing through the east and west points, is the prime vertical.



The ecliptic is divided into twelve parts, called signs, which receive their names from constellations lying near them. These divisions or signs are supposed to begin at that intersection of the celestial equator and ecliptic which is near the constellation Aries.

Great circles passing through the poles of the heavens are called *circles of declination;* and great circles passing through the poles of the ecliptic are called circles of latitude. Thus, let A Q represent a part of the celestial equator, A C a part of the ecliptic, A the first point of Aries, and therefore angle C A Q the obliquity of the ecliptic: let P be the pole of the heavens, or of the celestial equator, and P′ the pole of the ecliptic, then P X R is a circle of declination, and P′ X M is a circle of latitude.



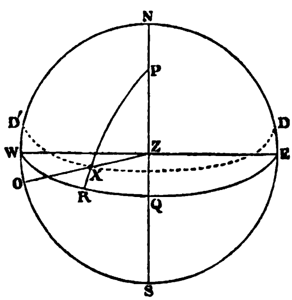
Parallels of declination and of *latitude* are small circles parallel respectively to the celestial equator and ecliptic.

The *declination* of a heavenly body is the arc of a circle of declination passing through its place in the celestial concave, intercepted between that place and the celestial equator: thus let X be the place of a heavenly body, then X R is its declination.

The *right ascension* of a heavenly body is the arc of the equator, intercepted between the first point of Aries and the circle of declination, passing through the place of the heavenly body in the celestial concave, measuring from the first point of Aries, eastward, from 0° to 360°; thus the arc A R is the right ascension of the heavenly body X.

In like manner, if a circle of latitude be drawn through any point X, in the celestial concave, the part of it between the point and the ecliptic is called the *latitude* of the point; and the arc of the ecliptic, extending eastward from the first point of Aries to the circle of latitude, is called the *longitude* of the point: thus the latitude of X is X M, and the longitude A M.

The *altitude* of a heavenly body is the arc of a circle of altitude passing through the place of the body intercepted between the place and the horizon. Thus, if Z O be a circle of altitude, and N W S E the horizon, then arc X O is the altitude of X.



The *azimuth*, or bearing of *a* heavenly body, is the arc of the horizon intercepted between the north and south points and the circle of altitude passing through the place of the body; or it is the corresponding angle at the zenith between the celestial meridian and the circle of altitude passing through the body: thus the arc S O or N O, or the angle N Z O, or S Z O, is the azimuth of X.

The *amplitude* of a heavenly body is the distance from the

east point at which it rises, or the distance from the west point at which it sets, the arcs or distances being measured on the horizon; thus suppose the heavenly body X to rise at D, and after describing the arc D X D′, to set at D′, then the amplitude of X is either D E or D′ W.

The *hour angle* of a heavenly body, is the angle at the pole between the celestial meridian and the circle of declination passing through the place of the body; thus, Z P X is the hour angle of X.

CHAPTER II.

ON TIME.

The Solar year, and Sidereal year.

15. A *solar year* is the interval between the sun's leaving the first point of Aries, and returning to it again.

A *sidereal year* is the interval between the sun's leaving a fixed point, as a star, and returning to that point again.

The equinoctial points have an annual motion of $50''{\cdot}1$, by which they are carried back to meet the sun in its apparent motion among the fixed stars, from west to east.

On this account a solar year is shorter than a sidereal year by the time the sun takes to describe $50''{\cdot}1$.

The length of the solar years is found to differ a little from each other, on account of certain irregularities in the sun's apparent motion, and that of the first point of Aries. The *mean length* of several solar years is therefore the one made use of in the common division of time, and called the *mean solar year*.

To find the length of the mean solar year.

16. By comparing observations made at distant periods, it was found that the sun had described $36000° 45' 45''$ of

longitude in 36525 days. Now in one solar year the sun separates from the first point of Aries 360°, taking into consideration its own apparent motion from west to east, and the actual motion of the first point of Aries in the opposite direction.

Let, therefore, $x =$ the length of a mean solar year;

$$\text{then, } 36000^\circ\,45'\,45'' : 360^\circ :: 36525^d : x$$

$$\therefore\ x = 365^d\,5^h\,48^m\,51^s{\cdot}6 = 365^d{\cdot}242264\,*$$

To find the length of the sidereal year.

17. Since the first point of Aries moves with a slow annual motion of about $50''{\cdot}1$ from east to west to meet the sun, the arc of the ecliptic described by the sun from the first point of Aries to the first point of Aries again, must be $360^\circ - 50''{\cdot}1 = 359^\circ\,59'\,9''{\cdot}9$, and this is the arc described by the sun in a mean solar year; but in a sidereal year the sun describes 360°; hence a sidereal is greater than a solar year by the time the sun takes to move over the arc of $50''{\cdot}1$.

Let $x =$ the length of the sidereal year.

$$\text{then sidereal year} : \text{mean solar year} :: 360^\circ : 360^\circ - 50''{\cdot}1$$

$$\text{or, sidereal year} : 365^d{\cdot}242264 :: 360^\circ : 359^\circ\,59'\,9''{\cdot}9$$

$$\therefore\ \text{sidereal year} = 365^d\,6^h\,9^m\,11^s{\cdot}5\ \dagger$$

The sidereal day, the apparent solar day, and the mean solar day.

18. The *sidereal day* is the interval between two successive transits of the first point of Aries over the same meridian. It begins when that point of Aries is on the meridian.

The *apparent solar day* is the interval between two successive transits of the sun's centre over the same meridian. It begins when that point is on the meridian.

* According to Bessel the formula for determining the length of the mean solar or tropical year is

$$365^d{\cdot}2422013 - {\cdot}00000006686 \times t$$

where t is the number of years since 1800.

† The length of a sidereal year according to Bessel is

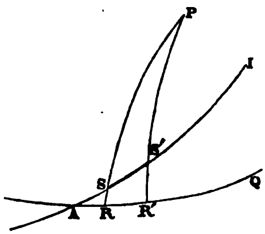
$$365^d{\cdot}256374322 = 365^d\,6^h\,9^m\,10^s{\cdot}7423 \text{ mean time.}$$

The length of an apparent solar day is variable from two causes:—

1st. From the variable motion of the sun in the ecliptic.

2nd. From the motion of the sun being in a circle inclined to the equator.

19. To explain briefly these causes of variation, let us suppose the two circles A Q, A I, to represent the celestial equator and ecliptic, and S S′ the arc described by the sun in one day. The angle at P, between the two circles of declination, is measured, not by the arc S S′ described by the sun, but by the arc R R′ of the equator. Now,



1st. The velocity or motion of the sun in the ecliptic is variable, on account of the earth moving in an elliptic orbit; it sometimes describes an arc of 57′ in a day; at other times the arc described is about 61′: this is one cause of the inequality in the length of the solar days.

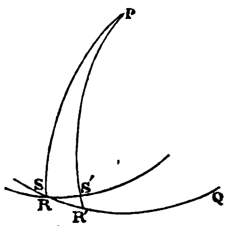


Fig. 2.

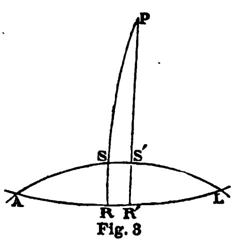


Fig. 3

2nd. But even supposing the arcs of the ecliptic described by the sun to be equal, yet the angles at P between the meridians as R P R′ (in the three figures) will not be so, since these angles are measured by the arc R R′ of the equator to which S S′ will be differently inclined according to the place of the sun in the ecliptic. At the equinoxes, or when the sun is at A, the arcs S S′ and R R′ will be inclined to each other at an angle of about 23° 27′ (see fig. 2). At the solstices they are parallel (see fig. 3). This is the second cause of the inequality.

20. To obtain a proper measure of time, we must proceed therefore as follows: an imaginary, or as it is called a *mean sun*, is supposed to move uniformly in the equator with the *mean* velocity of the true sun. A *mean solar day* may therefore be defined to be the interval between two successive transits of the mean sun over the same meridian. It begins when the mean sun is on the meridian.

To find the daily motion of the mean sun in the equator.

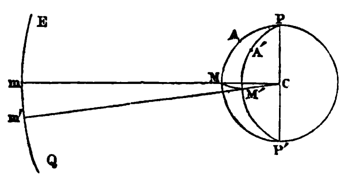
21. The mean solar year, or the time the sun takes to return again to the first point of Aries, has been found to be equal to $365^d{\cdot}2422$. Let us suppose the mean sun to describe the *equator* in this time, then we shall find its daily motion in the equator as follows:—Let x = daily motion,

$$365^d{\cdot}2422 : 1^d :: 360^\circ : x = 0^\circ{\cdot}9856472 = 59'\,8''{\cdot}33$$

or, the mean sun's daily motion in the equator from west to east is $59'\,8''{\cdot}33$.

To find the arc described by a meridian of the earth, in a mean solar day.

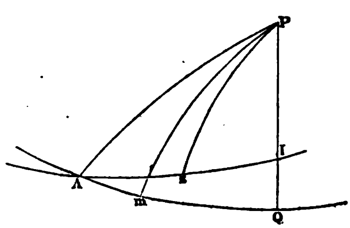
22. Let P A M P′ represent the meridian of a spectator A, drawn in some plane; as, for instance, that of the paper; E Q the celestial equator which must therefore be supposed at right angles to the paper. Suppose the mean sun to be at m on the meridian of A, and therefore in the plane of the paper; and let $m\,m'$ be the arc of the equator described by the mean sun in one day, namely, $59'\,8''{\cdot}33$. Now, let the earth be supposed to revolve about P P′, from west to east, until the meridian again passes through the mean sun, which has arrived at M′. Then the whole number of degrees described by the meridian of the spectator will evidently be one complete revolution, or 360° (by which it is again brought into the plane



of the paper), together with the arc $\text{M M}' = m\,m'$, or $59'\ 8''\cdot33$. Hence in a mean solar day a meridian describes $360^\circ\ 59'\ 8''\cdot33$.

Sidereal time, apparent solar time, and mean solar time.

23. *Sidereal* time is the angle at the pole of the heavens between the celestial meridian and a circle of declination passing through the first point of Aries, measuring from the meridian westward; thus, if P Q be the celestial meridian, A the first point of Aries, A Q the equator, then the angle Q P A is sidereal time.



24. *Mean solar time* is the angle at the pole between the celestial meridian, and a circle of declination passing through the mean sun, measuring from the meridian westward: thus let m be the mean sun considered as a point, then Q P m is mean solar time. Similarly, if S be the place of the true sun in the ecliptic A I, the angle Q P S measured from P Q westward is *apparent solar time.*

25. *The equation of time* is the difference in time between the places of the true and mean sun: thus, the angle m P S is the equation of time.

Sidereal clock, and mean solar clock.

26. A *sidereal clock* is a clock adjusted so as to go 24 hours during one complete revolution of the earth; that is during the interval of two successive transits of a fixed star: or supposing the first point of Aries to be invariable between two successive transits of the first point of Aries.

A *mean solar clock* is a clock adjusted to go 24 hours during one complete revolution of the mean sun; or while a sidereal clock is going $24^h\ 3^m\ 56^s\cdot555$.

CHAPTER III.

INTRODUCTORY OR PRELIMINARY RULES IN NAUTICAL ASTRONOMY.

Nautical day and Astronomical day.

27. The nautical or civil day begins at midnight and ends the next midnight. The astronomical day begins at noon (Art. 20) and ends at noon, and is later than the civil day by 12 hours. Again, in the astronomical day the hours are reckoned throughout from 0^h to 24^h; in the nautical day there are twice 12 hours, the first 12 hours being before noon, or before the commencement of the astronomical day (denoted by A.M., *ante meridiem*); the latter are afternoon, and distinguished by the letters P.M. (*post meridiem.*)

Rule I.

Given civil or nautical time at ship, to reduce it to astronomical time.

1. If the nautical time at ship be P.M., it will be also astronomical time, P.M. being omitted.

2. If the nautical time be A.M., add 12^h thereto, and put the day one back; thus—

(1.) April 27, at 4^h 10^m P.M. (civil) is April 27, at 4^h 10^m (astro.)
(2.) April 27, at 4 10 A.M. (civil) is April 26 at 16 10 (astro.)

EXAMPLES.

Reduce the following civil or nautical times to astronomical times.

	Civil times.					Astronomical times.		
(1.)	Sept. 10th,	4^h	10^m	P.M.	Ans.	Sept. 10th,	4^h	10^m
(2.)	June 3	2	42	A.M.	„	June 2	14	42
(3.)	July 1	6	18	A.M.	„	June 30	18	18
(4.)	Dec. 10	3	42	P.M.	„	Dec. 10	3	42

Rule II.

Given astronomical time at the ship, to reduce it to civil or nautical time.

1. If the astronomical time is less than 12 hours, it will also be nautical time P.M.

2. If the astronomical time be greater than 12 hours, reject 12 and put the day one forward; the result will be civil time A.M.; thus—

(1.) April 27, at $4^h\ 10^m$ (astro.) is April 27, at $4^h\ 10^m$ P.M. (civil).
(2.) April 27, at 16 10^m (astro.) is April 28, at 4 10 A.M. (civil).

EXAMPLES.

Reduce the following astronomical times to nautical or civil times.

	Astronomical times.			Civil times.			
(5.)	Sept. 10th,	4^h	32^m	Ans. Sept. 10th,	4^h	32^m	P.M.
(6.)	July 5	16	32	„ July 6	4	32	A.M.
(7.)	July 10	18	42	„ July 11	6	42	A.M.
(8.)	Dec. 21	23	59	„ Dec. 22	11	59	A.M.

Given the time at Greenwich, to find the time at the same instant at any other place, and the converse.

28. To find the time at any place, as Greenwich, corresponding to a given time at any other place, or the converse, we must remember that since the earth revolves through 360° in 24 hours, from west to east, or 15° in 1 hour, and therefore through 1° in 4 minutes, or 1′ of arc in 4 seconds of time, at a place 15° to the eastward of a spectator the sun will be on the meridian 1 hour before, and at a place 15° to the westward, the sun will be on the meridian 1 hour later than at the place of the spectator: hence, when it is 10 o'clock at a given place, it will at the same instant be 11 o'clock at a place 15° to the eastward, and 9 o'clock at a place 15° to the westward. If, therefore, the longitude of a place is known, that is, the number of degrees it is to the east or west of Greenwich, we can readily tell what time it

is at the place corresponding to a given time at Greenwich, and the converse. To find the time at Greenwich, corresponding to any given time at a place, is required in almost every nautical problem; and even if the longitude of the place and time are only known nearly, the approximate true time at Greenwich, deduced from the estimated longitude and time at the place, is an important element in nautical astronomy. The time at Greenwich, obtained in this manner, is called an approximate Greenwich date, or more frequently *the Greenwich date.*

To find the Greenwich date, we shall require the following rules for reducing degrees into time, and the converse.

Rule III.

To reduce degrees into time.

(1.) Divide the degrees by 15, the quotient is hours.

(2.) Multiply the remaining degrees, if any, by 4; the result is minutes in time.

(3.) Divide the minutes in arc by 15; the quotient is minutes in time.

(4.) Multiply the remaining minutes of arc, if any, by 4; the result is seconds of time.

(5.) Divide the seconds in arc by 15; the quotient is seconds in time, carried to decimals if necessary. The sum will be the arc in time.

EXAMPLE.

Reduce $34^\circ\ 44'\ 34''$ into time.

$$
\begin{array}{rcrrr}
34^\circ & = & 2^h & 16^m & 0^s \\
44' & = & & 2 & 56 \\
34'' & = & & & 2{\cdot}26 \\
\hline
\therefore\ 34^\circ\ 44'\ 34'' & = & 2 & 18 & 58{\cdot}26
\end{array}
$$

TABLE

To reduce degrees into time, and the converse.

1′ = 0m 4s	21′ = 1m 24s	41′ = 2m 44s	1° = 0h 4m	10° = 0h 40m
2′ = 0 8	22′ = 1 28	42′ = 2 48	2° = 0 8	20° = 1 20
3′ = 0 12	23′ = 1 32	43′ = 2 52	3° = 0 12	30° = 2 0
4′ = 0 16	24′ = 1 36	44′ = 2 56	4° = 0 16	40° = 2 40
5′ = 0 20	25′ = 1 40	45′ = 3 0	5° = 0 20	50° = 3 20
6′ = 0 24	26′ = 1 44	46′ = 3 4	6° = 0 24	60° = 4 0
7′ = 0 28	27′ = 1 48	47′ = 3 8	7° = 0 28	70° = 4 40
8′ = 0 32	28′ = 1 52	48′ = 3 12	8° = 0 32	80° = 5 20
9′ = 0 36	29′ = 1 56	49′ = 3 16	9° = 0 36	90° = 6 0
10′ = 0 40	30′ = 2 0	50′ = 3 20	10° = 0 40	100° = 6 40
11′ = 0 44	31′ = 2 4	51′ = 3 24	11° = 0 44	110° = 7 20
12′ = 0 48	32′ = 2 8	52′ = 3 28	12° = 0 48	120° = 8 0
13′ = 0 52	33′ = 2 12	53′ = 3 32	13° = 0 52	130° = 8 40
14′ = 0 56	34′ = 2 16	54′ = 3 36	14° = 0 56	140° = 9 20
15′ = 1 0	35′ = 2 20	55′ = 3 40	15° = 1 0	150° = 10 0
16′ = 1 4	36′ = 2 24	56′ = 3 44	16° = 1 4	160° = 10 40
17′ = 1 8	37′ = 2 28	57′ = 3 48	17° = 1 8	170° = 11 20
18′ = 1 12	38′ = 2 32	58′ = 3 52	18° = 1 12	180° = 12 0
19′ = 1 16	39′ = 2 36	59′ = 3 56	19° = 1 16	190° = 12 40
20′ = 1 20	40′ = 2 40	60′ = 4 0	20° = 1 20	200° = 13 20

Or, thus, by means of the Table to the nearest minute.

$$\begin{aligned} 30^\circ &= 2^h\ 0^m \\ 4^\circ &= \quad 16 \\ 44' &= \quad\ \underline{3} \text{ nearly.} \\ \therefore\ 34^\circ\ 44'\ 34'' &= 2\ 19 \text{ nearly.*} \end{aligned}$$

In some nautical tables, the angles in the log. sine table are given both in arc and time. The reduction from degrees to hours, and the converse, is by means of such a table readily made.

EXAMPLES.

Reduce the following arcs into time.

(9.) $84^\circ\ 42'\ 30''$ Ans. $5^h\ 38^m\ 38^s$

(10.) 96 10 45 „ 6 24 43

* The table is computed to the nearest minute of arc; when seconds are to be reduced (which is seldom required) the student must proceed as pointed out in the preceding example and rule.

(11.) $108° 24' 22''$	Ans.	$7^h 13^m 55^s{\cdot}4$
(12.) 178 48 45	„	11 55 15
(13.) 140 32 10	„	9 22 8·66
(14.) 230 32 10	„	16 2 8·66

Rule IV.

To reduce time into degrees.

1. Multiply the hours by 15; the result is degrees.

2. Divide the minutes in time by 4; the quotient is degrees.

3. Multiply the minutes remaining, if any, by 15; the result is minutes of arc.

4. Divide the seconds of time by 4; the quotient is minutes of arc.

5. Multiply the seconds (and parts of seconds) remaining, if any, by 15; the result is seconds of arc.

The sum will be the arc in degrees.

EXAMPLE.

Reduce $2^h 18^m 58^s{\cdot}26$ into degrees.

$$\begin{array}{rcr} 2^h & = & 30° \ 0' \ 0'' \\ 18^m & = & 4 \ 30 \ 0 \\ 58^s{\cdot}26 & = & 14 \ 33{\cdot}9 \\ \hline \therefore \ 2^h 18^m 58^s{\cdot}26 & = & 34 \ 44 \ 33{\cdot}9 \end{array}$$

Or thus, by means of the table, to the nearest minute.

$$2^h 18^m 58^s{\cdot}26 = 2^h 19^m \text{ nearly.}$$

$$\begin{array}{rcr} 2^h & = & 30° \ 0' \\ 16^m & = & 4 \ 0 \\ 3 & = & 0 \ 45 \\ \hline 2^h 19^m & = & 34 \ 45 \end{array}$$

EXAMPLES.

Find the arcs corresponding to the following times,

(15) $3^h 52^m 4^s$	Ans.	$58° 1' 0''$
(16) 17 8 22	„	227 5 30

(17)	8^h	17^m	$15{\cdot}5^s$	Ans.	124°	18′	52″·5
(18)	12	14	16·75	„	183	34	11·25
(19)	9	13	8	„	138	17	0
(20)	15	17	18·4	„	229	19	36

Rule V.

To find the Greenwich date, having given time at ship and the longitude.

1. Express the time at the ship astronomically (p. 65).

2. Reduce the longitude into time, and put it under ship time (p. 68).

3. If west longitude, add longitude in time to ship time; the sum, if less than 24 hours, will be the time at Greenwich, or the Greenwich date on the same day as at the ship.

But if the sum be greater than 24 hours, reject 24 hours; the result will be the Greenwich date on the day following the ship date.

If east longitude, subtract longitude in time from ship time, the remainder will be the Greenwich date. If the longitude in time is greater than the ship time, 24 hours must be added to the ship time before subtraction is made, and the Greenwich date will be the remainder on the day preceding the ship date.

EXAMPLE.

June 10th, at 6^h 10^m P.M., in longitude 32° 42′ W., required the time at Greenwich, or the Greenwich date to the nearest minute.

Ship, June 10th, at	6^h	10^m	
long. in time . .	2	11	W.
Gr. June 10th . .	8	21	

July 12th, at 4^h 5^m A.M., in long. 63° 45′ W., required the Greenwich date.

Ship, July 11th, at	16^h	5^m	
long. in time . .	4	15	W.
Gr. July 11th . .	20	20	

EXAMPLES.

Required the Greenwich date in each of the following examples.

	Ship times.							Answers, Greenwich dates.		
(21)	Mar. 7, at	3^h	15^m	A.M.	Long. 15°	45′E.	.	Mar. 6, at	14^h	12^m
(22)	Mar. 15 „	10	35	P.M.	„ 43	5 E.	.	Mar. 15 „	7	43
(23)	May 12 „	4	30	A.M.	„ 45	50 W.	.	May 11 „	19	33
(24)	May 9 „	5	16	P.M.	„ 90	35 E.	.	May 8 „	23	14
(25)	May 5 „	11	30	P.M.	„ 55	47 W.	.	May 5 „	15	13
(26)	May 20 „	10	25	A.M.	„ 150	15 W.	.	May 20 „	8	25

Second Method of finding a Greenwich Date.

The Greenwich date is more correctly found by means of a chronometer, whose error on Greenwich mean time is known.

Rule VI.

To the time shown by chronometer, apply its error on Greenwich mean time; adding if error is slow, and subtracting if error is fast, on Greenwich mean time; the result is the Greenwich date in mean time. Sometimes 12 hours must be added to this result, and the day put one back. This uncertainty may be removed by getting an approximate Greenwich date in the usual way by means of ship mean time and the estimated longitude; if the difference between the Greenwich dates found by the two methods is nearly 12 hours, then the Greenwich date *by chronometer* must be increased by 12 hours, and the day put one day back, if necessary, so as to make the two dates agree both in the day and hour nearly.

The following examples will remove any doubt as to putting the day one back, or not.

EXAMPLE.

July 10th, 1853, at 6^h 34^m P.M. mean time nearly, in longitude 60° W., a chronometer showed 10^h 42^m 3^s, its error on Greenwich mean time being 2^m 10^s fast; required mean time at Greenwich, or the Greenwich date.

Greenwich date by chronometer.				Greenwich date approximately.		
July 10th, chro. .	10^h	42^m	3^s	Ship, July 10th	6^h	34^m
Error on G. M. T.		2	10	long. in time .	4	0 W.
Gr. July 10th . .	10	39	53	Gr. July 10th	10	34

As these two results come out near to each other, the correct Greenwich date is July 10th, $10^h\ 39^m\ 53^s$.

EXAMPLE 2.

Aug. 3rd, 1853, at $5^h\ 42^m$ P.M. mean time nearly, in long. by account 150° 30′ W., a chronometer showed $3^h\ 23^m\ 15^s$, and its error on Greenwich mean time was $10^m\ 10^s{\cdot}4$ slow; required the Greenwich date.

Greenwich date by chronometer.				Greenwich date approximately.		
Aug. 3rd, at .	3^h	23^m	15^s	Ship, Aug. 3rd.	5^h	42^m
Error . . .		10	10·4	long. in time .	10	2 W.
Aug. 3rd . .	3	33	25·4	Gr. Aug. 3rd .	15	44
Add	12					
Gr. Aug. 3rd . .	15	33	25·4			

In this example 12 hours must be added to the Greenwich date by chronometer, without putting the day one back.

EXAMPLE 3.

March 10th, 1853, at $2^h\ 10^m$ A.M. mean time nearly, in longitude 20° 42′ E., a chronometer showed $0^h\ 2^m\ 50^s$, and its error on Greenwich mean time was $45^m\ 16^s$ slow; required the Greenwich date.

Greenwich date by chronometer.				Greenwich date approximately.		
Mar. 10th . .	0^h	2^m	50^s	Ship, Mar. 9th .	14^h	10^m
Error . . .		45	16	long. in time .	1	23 E.
Mar, 10th . .	0	48	6	Gr. Mar. 9th .	12	47
Add	12					
Gr. Mar. 9th .	12	48	6			

In this example 12 hours must be added, and the day put one back, to bring the chronometer Greenwich date more nearly alike to the estimated Greenwich date.

Find accurately the Greenwich dates in the following examples.

	M. T. nearly.	Long.	Chro. show.	Err. of chro.	Answers.
(27)	Aug. 10 1^h 20^m P.M.	35° 42′ W.	4^h 2^m 10^s	18^m $45{\cdot}4^s$ fast	10 8^h 43^m $24{\cdot}6^s$
(28)	July 18 3 42 A.M.	150 50 W.	1 30 0	10 50·6 slow	13 1 40 50·6
(29)	June 10 10 42 P.M.	42 0 E.	7 44 10	8 12·0 slow	10 7 52 22
(30)	June 19 6 42 A.M.	50 50 W.	10 14 15	12 8·7 fast	18 22 2 11·3
(31)	Sept. 8 10 42 A.M.	19 15 E.	9 10 45	12 15·8 slow	2 21 23 0·3
(32)	Dec. 30 11 45 P.M.	110 35 W.	7 10 30	9 5·0 fast	30 19 1 25

CHAPTER IV.

EXPLANATION AND USE OF THE NAUTICAL ALMANAC.

THE NAUTICAL ALMANAC contains the declination, right ascension, &c., of the principal heavenly bodies, for certain fixed times at Greenwich. The declination and right ascension of the sun and planets are given for every day at 0^h 0^m 0^s; for the moon, for every hour at Greenwich. To obtain these quantities for any other time, we may either apply the common rules of proportion; or—which is in most cases the simplest method—make use of certain tables computed for the purpose, called tables of *proportional logarithms*. The tables of proportional logarithms contained in most collections of nautical tables are the following:—

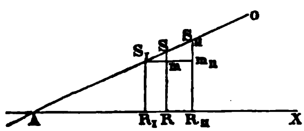
1. The proportional logarithms (properly so called).
2. The Greenwich date proportional logarithm of the sun.
3. The Greenwich date proportional logarithm of the moon.
4. The logistic logarithms.

Their use will be best seen in the following rules and examples.

Rules for taking out of the Nautical Almanac the sun's declination, &c.

The sun's change of declination, and the method of determining its amount at any given time, may be illustrated by means of a figure.

Let A X be the celestial equator, A O the ecliptic, $S_{,}$ and $S_{,,}$ the places of the sun on the 27th and 28th April, 1846, at Greenwich mean noon; then at 10 o'clock on the 27th it must be at some intermediate point, as at S; and $S_{,}\,R_{,}$ $S_{,,}\,R_{,,}$ are the declination of the sun on the 27th and 28th at noon, and S R the declination at the required time. Draw $S_{,}\,m_{,,}$ parallel to the equator, then $S_{,,}\,m_{,,}$ is the change of the sun's declination in 24 hours, and S m the change in 10 hours, and if we suppose the sun's motion in the ecliptic during the 24 hours to be equable, and $S_{,,}\,m_{,,}$ a few minutes only, so that the lines drawn in the figure may be considered as straight lines, we have this proportion:



$$S\,m : S_{,,}\,m_{,,} :: 10^h : 24^h$$

Hence S m is easily found, since $S_{,,}\,m_{,,}$, the change in 24 hours, is known by means of the Nautical Almanac. And in a similar manner may be explained the method of taking out, for any required time at Greenwich, the other quantities given in the Nautical Almanac.

Rule VII.

To take out the sun's declination.

First method. By proportional logarithms.

1. Get a Greenwich date; thus, put down the ship mean time expressed astronomically.

2. Under which put the longitude in time.

3. Add in west, subtract in east longitude (adding or subtracting 24 hours, according to the Rule I, p. 71).

4. Take out of the Nautical Almanac the sun's declination for two consecutive noons between which the Greenwich date lies.

5. Take the difference of the declinations when their names are alike; but when the names of the declinations are unlike, take their sum; thus finding the change of declination in 24 hours.

6. Add together Greenwich date logarithm for the sun and proportional logarithm of the change in 24 hours; the result is the proportional logarithm of change of declination for the given time, which take from the Tables and apply to the declination at first noon, either by subtracting or adding it, according as the declination is seen to be decreasing or increasing.

Rule VIII.

Second method. By hourly differences.

Another method of taking out the sun's declination, is to make use of the hourly changes of declination given in the Nautical Almanac.

1. Find a Greenwich date as before.

2. Take out of the Nautical Almanac the declination at noon of the Greenwich date, and put down a little to the right thereof the difference for one hour found in page 1 of the Nautical Almanac. Multiply this quantity by the hours in Greenwich date, and the fractional parts of the hour if necessary, the product will be the change of declination in the time from noon; apply this, reduced to minutes and seconds, to the declination taken out, adding it if the declination is seen to be increasing, and subtracting if decreasing. The result is the declination of the sun at the time required.*

* The corrections of the quantities taken out of the Nautical Almanac are often made *by inspection:* the results thus obtained are generally considered sufficiently correct.

EXAMPLES.

1. March 2, 1853, at $4^h\ 23^m$ P.M., mean time, in long. 32° 42′ W., required the sun's declination.

Ship, March 2 . . .	4^h 23^m	
Long. in time . . .	2 11	W.
Greenwich, March 2 .	6 34	

Sun's declination.

March 2	7° 7′ 0″ S.
„ 3	6 44 2 S.
	22 58
Greenwich date ⊙ . .	56287
Prop. log. 22′ 58″ . . .	89417
	1·45704
Cor.	6′·17″
Sun's declination . .	7 0 43 S.

Or thus, by hourly difference.

March 2 . . .	7° 7′ 0″ S.
Hourly diff. .	57″·4 decreasing
	6
	344 ·4 diff. for 6
30^m . $\frac{1}{2}$	28 ·7
3 . $\frac{1}{10}$	2 ·9
1 . $\frac{1}{3}$	·9
	60) 376 ·9
Cor. . . .	6 16·9
Or . . .	6 17
March 2 . . .	7° 7 0 S
Sun's decl. . .	7 0 43 S

2. March 21, 1851, at $10^h\ 42^m$ A.M. mean time, in long. 15° 30′ W., required the sun's declination.

Ship, March 20 . . .	22^h 42^m	
Long. in time . . .	1 2	W.
Greenwich, March 20 .	23 44	

20	0° 16′ 42″ S.
21	0 6 59 N.
	23 41
	00485
	88083
	88568
Cor.	23 25
Sun's decl. . .	0 6 43 N

(33.) May 20, 1850, at 5^h 30^m P.M. mean time, in long. 95° 30′ E., required the sun's declination.

Ans. 19° 57′ 22″ N.

(34.) May 16, 1850, at 7^h 50^m A.M. mean time, in long. 120° 0′ E., required the sun's declination.

Ans. 18° 57′ 48″ N.

(35.) March 23, 1850, at 3^h 20^m P.M. mean time, in long. 9° 0′ W., required the sun's declination.

Ans. 1° 3′ 53″ N.

Elements from Nautical Almanac.

May 19	19°	45′	6″ N.	.	May 20	19°	57′	49″ N.
May 15	18	50	52 N.	.	May 16	19	4	55 N.
March 23	1	0	6 N.	.	March 24	1	23	43 N.

Rule IX.

To take out the equation of time.

1. Get a Greenwich date.

2. Take out the equation of time for two consecutive noons between which the Greenwich date lies, and take their difference.

3. Add together the Greenwich date logarithm for sun and proportional logarithm of difference: the sum is the proportional logarithm of correction, which find from the table, and apply it with its proper sign to the equation of time first taken out; the result is the equation of time required.

Or thus, by hourly differences.

1. Take out the equation of time for the noon of Greenwich date and the hourly difference opposite thereto.

2. Multiply hourly difference by the hours of the Greenwich date, and, if great accuracy is required, by the fractional parts of hour in the Greenwich date; the result will be the correction to be applied with its proper sign to the equation of time taken out.

EXAMPLE.

July 12, 1853, at $5^h\ 8^m$ A.M. mean time nearly, in long. 160° W., required the equation of time.

Ship, July 11	17^h	8^m
Long. in time	10	40 W.
Greenwich, July 11 . .	27	48
Greenwich, July 12 . .	3	48

Equation of time.		Or thus, by hourly difference.	
12	$5^m\ 15^s{\cdot}7$	Diff. for 1 hour . .	$0^s{\cdot}308$ incr.
13	5 23 ·1		3
	7 ·4		0 ·924
Greenwich d. log. sun .	0·80043	30 ½	·154
Prop. log. 7″·4 . . .	3·16419	15 ¼	·077
Prop. log. cor. . . .	3·96462	Cor.	1 ·155
Cor.	1·2	Or, . .	$1^s{\cdot}2$
Equation required . .	5 16 ·9	12.	5^m 15 ·7
		Equation required	5 16 ·9

Find the equation of time in the following examples:—

(36.)	March 2, 1853, at	6^h	10^m P.M.	mean time in long.	38°	42′ W.
(37.)	„ 16 „ „	5	42 A.M.	„ „	152	45 W.
(38.)	„ 29 „ „	10	42 A.M.	„ „	87	8 E.

Elements from Nautical Almanac.

							Diff. 1 hour.
Eq. of time March	2	12^m	$22^s{\cdot}1$.	March 3	12^m	$9^s{\cdot}3$.	$0^s{\cdot}53$
„ „	16	8	48 ·4 .	„ 17	8	30 ·8 .	0 ·72
„ „	28	5	9 ·0 .	„ 29	4	50 ·4 .	0 ·77

Answers to (36), (37), (38):—

$12^m\ 17^s{\cdot}4$ $8^m\ 45^s{\cdot}5$ $5^m\ 3^s{\cdot}7$

Rule X.

To take out the moon's semidiameter and horizontal parallax.

The moon's semidiameter and horizontal parallax are put down in the Nautical Almanac for every mean noon and

mean midnight at Greenwich: to find these quantities for any other time we may proceed as follows:—

First. To find the moon's semidiameter.

1. Get a Greenwich date.

2. Take out of the Nautical Almanac the moon's semidiameter for the two times between which the Greenwich date lies, and take the difference. To the Greenwich date logarithm for moon add the proportional logarithm of the difference just found; the result will be the proportional logarithm of an arc, which being found and added to the semidiameter first taken out, or subtracted therefrom (according as the semidiameter is increasing or decreasing), will be the semidiameter at the given time.

Second. To find the moon's horizontal parallax.

Proceed in a similar manner to that pointed out above for finding the moon's semidiameter.

EXAMPLES.

1. August 3, 1853, at $4^h\ 10^m$ P.M. mean time nearly, in long. 48° 42′ W., required the moon's semidiameter and horizontal parallax.

Ship, August 3 . .	4^h	10^m
Long. in time . . .	3	15 W.
Greenwich, Aug. 3 .	7	25

Moon's semidiameter.			Moon's horizontal parallax.		
August 3, at noon .	15′	6″·6	August 3, at noon .	55′	20″·6
„ mid. .	15	10 ·6	„ mid. .	55	35 ·3
		4 ·0			14 ·7
Gr. d. log. for moon.		·18064	Gr. d. log. for moon.		·18064
Prop. log. for 4″·0 .		3·43136	Prop. log. for 14″·7 .		2·86611
Prop. log. cor. . .		3·61200	Prop. log. cor. . .		3·04675
Cor.		2 ·6	Cor.		9 ·7
Required semi. . .	15	9 ·2	Required hor. par. .	55	30 ·3

2. July 14, 1853, at $6^h\ 42^m$ A.M. mean time nearly, in long. 30° W., required the moon's semidiameter and horizontal parallax.

Ship, July 13	18^h	42^m
Long. in time	2	0 W
Greenwich, July 13 . .	20	42

Moon's semidiameter.		Moon's horizontal parallax.	
July 13, mid. . . .	16′ 2″·7	July 13, mid. . . .	58′ 45″·8
„ 14, noon . . .	16 7 ·5	„ 14, noon . . .	59 3 ·5
	4 ·8		17 ·7
Gr. d. log. moon for $8^h 42^m$*	·13966	Gr. d. log. moon for $8^h 42^m$	13966
Prop. log. 4″·8 . . .	3·35218	Prop. log. 17″·7 . .	2·78545
Prop. log. cor. . . .	3·49184	Prop. log. cor. . .	2·92511
Cor.	3 ·5	Cor.	12 ·8
Required semi. . . .	16 6 ·2	Required hor. par. .	58 58 ·6

Find the moon's semidiameter and horizontal parallax in the following examples:—

(39.) March 2, 1853 . . at 6^h 42^m P.M. . . in long. 100° 0′ W.
(40.) „ 14 „ . . at 3 30 A.M. . . „ 120 0 W.
(41.) „ 24 „ . . at 10 10 P.M. . . „ 60 42 E.

Elements from Nautical Almanac.

Moon's semidiameter.			Moon's horizontal parallax.			Answers.		
March 2, mid.	16′	5″·1	Mid.	58′	54″·7	Semi.	16′	4″·7
„ 3, noon	16	2 ·1	Noon	58	43 ·6	H. P.	58	53 ·4
March 13, mid.	14	48 ·9	Mid.	54	15 ·7	Semi.	14	47 ·9
„ 13, noon	14	47 ·8	Noon	54	11 ·5	H. P.	54	11 ·7
March 24, noon	16	18 ·7	Noon	59	44 ·6	Semi.	16	21 ·1
„ 24, mid.	16	23 ·7	Mid.	60	1 ·8	H. P.	59	53 ·1

* When the Greenwich date exceeds 12 hours, as in this example, look out the Greenwich date logarithm moon for the excess of the Greenwich date above 12 hours. It is better, however, in examples of this kind, where the differences are small, to estimate the correction *at sight*, without using logarithms; after some experience this is easily done; the above method, however, by means of logarithms, should be adopted by beginners.

Rule XI.

To take out the sun's right ascension.

1. Get a Greenwich date.

2. Take out the right ascension for two consecutive noons between which the Greenwich date lies, and take their difference.

3. Add together the Greenwich date logarithm for sun and proportional logarithm of difference; the sum will be the proportional logarithm of correction to be added to the right ascension for noon of Greenwich date.

EXAMPLE.

July 13, 1853, at 6^h 31^m A.M. mean time nearly, in long. 172° 10′ W., required the sun's right ascension.

Ship, July 12		18^h	31^m	
Long. in time		11	29	W.
July 12		30	0	
Greenwich, July 13 . .		6	0	
Sun's right-ascen. July 13 .		7°	30′	30″
„ „ 14 .		7	34	33
			4	3
	·60206			
	1·64782			
	2·24988		1	1 Cor.
Sun's right ascen. required .		7	31	31

Find the sun's right ascension in the following examples:—

(42.) March 11, 1853, at 6^h 42^m P.M. mean time, long. 42° 41′ W.
(43.) „ 21 „ „ 10 10 A.M. „ „ 100 41 E.
(44.) „ 21 „ „ 0 0 „ 142 14 W.

Elements from Nautical Almanac.

Sun's right asc. March 11	23^h	26^m	$26^s·3$	March 12	23^h	30^m	$6^s·6$
„ „ 20	23	59	20·0	„ 21	0	2	58·2
„ „ 21	0	2	58·2	„ 22	0	6	36·4

Answers to (42), (43), (44):—

23^h 27^m $53^s·3$ 0^h 0^m $10^s·0$ 0^h 4^m $24^s·2$

Rule XII.

To take out the moon's declination and right ascension.

The moon's declination and right ascension are recorded in the Nautical Almanac for the beginning of every hour of mean time at Greenwich. To find them for any other time we may proceed as follows:—

First. To find the moon's declination for any given time.

1. Get a Greenwich date.
2. Take out of the Nautical Almanac the moon's declination for two consecutive hours between which the Greenwich date lies, and take the difference.
3. Add together the logistic logarithm of minutes in Greenwich date and proportional logarithm of difference, the sum will be the proportional logarithm of correction, which take from the table and apply it to the declination for the hour of Greenwich date, adding or subtracting according as the declination is seen to be increasing or decreasing. The result is the declination required.

Second. To take out the moon's right ascension.

Proceed in a similar manner to that pointed out above for finding the moon's declination.

EXAMPLES.

January 24, 1852, at $5^h\ 10^m$ P.M. mean time, in long. 60° 10′ W., find the moon's right ascension and declination.

Ship, January 24, at . .	5^h	10^m	
Long. in time	4	1	W.
Greenwich, January 24 .	9	11	

Moon's right ascension.

Jan. 24, at	9^h . .		23^h	9^m	16^s
„	10 . .		23	11	11
				1	55
		·73676			
		1·97273			
		2·70949		0	21
			23	9	37

Moon's declination.

Jan. 24, at	9^h . .		10°	14′	23″	S.
„	10 . .		10	4	5	S.
				10	18	
		·73676				
		1·24244				
		1·97920		1	53	
			10	12	30	S.

(45.) June 2, 1852, at 2^h 30^m P.M. mean time, in long. 53° 15′ W., find the moon's right ascension and declination.

Ans., Right ascen. 17^h 11^m 53^s
Declination. 21° 15′ 54″ S.

(46.) Sept. 7, 1852, at 4^h 15^m A.M. mean time, in long. 56° 30′ E., find the moon's right ascension and declination.

Ans., Right ascen. 5^h 5^m 17^s
Declination 21° 17′ 12″ N.

(47.) July 10, 1853, at 9^h 30^m A.M. mean time, in long. 44° 20′ W., find the moon's right ascension and declination.

Ans., Right ascen. 10^h 36^m 34^s
Declination 14° 14′ 32″ N.

Elements from Nautical Almanac.

	Moon's right ascension.				Moon's declination.			
June 2, at 6^h	17^h	11^m	45^s	. . .	21°	15′	36″	S.
„ 7	17	14	17	. . .	21	21	34	S.
Sept. 6, at 12	5	4	13	. . .	21	14	34	N.
„ 13	5	6	22	. . .	21	20	1	N.
July 10, at 0	10	35	38	. . .	14	19	42	N.
„ 1	10	37	43	. . .	14	8	13	N.

Rule XIII.

To take out the right ascension of the mean sun (called in the Nautical Almanac sidereal time).

The right ascension of the mean sun, or the sidereal time at mean noon, is given in the Nautical Almanac for every day at mean noon. To find it for any other time we may proceed as in the rule for finding the right ascension of the apparent or true sun; but as the motion of the mean sun is uniform throughout the year (the motion in every 24 hours being 3^m $56^s{\cdot}555$), the change in any given number of hours, minutes, and seconds is more easily found by means of a table. This table is given in the Nautical Almanac, and

may be sought for in the Index under the title of "Time Equivalents, table of."

EXAMPLE.

July 23, 1853, at $2^h\ 42^m$ P.M. in long. 80° 42′ E., required the right ascension of the mean sun.

Ship, July 23 . . .	2^h	42^m
Long. in time . . .	5	23 E.
Greenwich, July 22 .	21	9

Right ascension mean sun.

July 22	8^h	0^m	35^s
„ 23	8	4	32
		3	57
·05490			
1·65868			
1·71358		3	29
Right asc. mean sun	8	4	4

Or thus, by table.

July 22	8^h	0^m	35^s
Cor. for 21^h . . .		3	27
„ 9^m . . .			1·5
	8	4	3·5

Find the right ascension of mean sun (called in the Nautical Almanac sidereal time) in the following examples:—

(48.)	March 2, 1853,	at	10^h	42^m	P.M.	mean time in long.	48°	10′ W.
(49.)	„ 15 „	„	6	6	A.M.	„	100	0 W.
(50.)	„ 21 „	„	10	10	P.M.	„	100	0 E.

Elements from Nautical Almanac and answers.

Sidereal time March 2, at noon,	22^h	40^m	$44^s·9$.	Ans.	22^h	43^m	$2^s·0$
„ „ 15 „	23	32	0·1 .	„	23	32	7·6
„ „ 21 „	23	55	39·4 .	„	23	56	13·9

Rule XIV.

To take out the lunar distances for any given time at Greenwich.

1. Get a Greenwich date.

2. Find two consecutive distances in the Nautical Almanac at times between which the Greenwich date lies. Take the difference of the distances. To the proportional logarithm of the excess of the Greenwich date above the first of the times taken from the Nautical Almanac add proportional

logarithm of difference of distances; the sum will be the proportional logarithm of an arc; which arc being applied to the distance at first time with its proper sign will be the distance required.

EXAMPLE.

September 24, at $6^h\ 10^m$ P.M. mean time nearly, in long. 60° 15′ W., required the distance of Aldebaran from the moon.

Ship, Sept. 24	6^h	10^m
Long. in time	4	1 W.
Greenwich, Sept. 24th .	10	11

			Distance of Aldebaran.		
	At 9^h	. .	18°	57′	35″
	12	. .	20	23	37
Proportional log. . .	32061	. .	1	26	2
Prop. log. $1^h\ 11^m$ . .	40401				
Cor.	72462	. .	0	33	56
Dist. of Aldebaran at $6^h\ 10^m$ P.M. .			19	31	31

Required the distance of the moon from certain stars in the following examples :—

(51.) Jan. 24, at $4^h\ 30^m$ P.M. mean time nearly, in long. 30° 30′ E., required the distance of Regulus from the moon.
Ans., 69° 33′ 6″.

(52.) May 20, at $6^h\ 20^m$ A.M. mean time nearly, in long. 40° 0′ E., required the distance of α Pegasi from the moon.
Ans., 56° 59′ 7″.

(53.) June 10, at $9^h\ 40^m$ P.M. mean time nearly, in long. 32° 45′ W., required the distance of α Aquilæ from the moon. Ans., 70° 32′ 35″.

(54.) July 2, at $7^h\ 20^m$ A.M. mean time nearly, in long. 30° 0′ E., required the distance of Jupiter from the moon.
Ans., 54° 16′ 52″.

(55.) Sept. 19, at $10^h\ 30^m$ A.M. mean time nearly, i-

long. 63° 15′ E., required the distance of Aldebaran from the moon. Ans., 72° 0′ 51″.

(56.) Dec. 15, at 2^h 0^m P.M. mean time nearly, in long. 19° 40′ E., required the distance of Pollux from the moon.
Ans., 58° 56′ 47″.

Elements from Nautical Almanac.

Distance of	Regulus	at noon	68°	11′	17″	. at	3^h	69°	50′	50″
,,	α Pegasi	,, 15^h	57	17	6	. ,,	18	55	56	9
,,	α Aquilæ	,, 9	75	56	46	. ,,	12	74	28	9
,,	Jupiter	,, 15	55	41	18	. ,,	18	53	52	44
,,	Aldebaran	,, 18	72	9	14	. ,,	21	70	40	29
,,	Pollux	,, noon	58	32	51	. ,,	3	60	17	54

In the rule for finding the longitude by lunar observations, we have to ascertain the *true* distance of the moon from some heavenly body at the time of observation. If the heavenly body is one whose distance is recorded in the Nautical Almanac for every three hours, we may find the mean time at Greenwich corresponding to the true distance computed for the time of observation as follows:—

Rule XV.

To find the time at Greenwich corresponding to a given distance of a heavenly body from the moon.

1. Under the given distance put down the two computed distances of the same heavenly body found in the Nautical Almanac between which the given true distance lies.

2. Take the difference between the first and second, and also between the second and the third.

3. From the proportional logarithm of the first difference subtract the proportional logarithm of the second difference, the sum is the proportional logarithm of the additional time to be added to the hours of the distance first taken out of the Nautical Almanac; the result is the mean time at Greenwich corresponding to the given distance.

EXAMPLES.

November 22, 1853, the true distance of Saturn from the moon was found to be 77° 52′ 45″, required Greenwich mean time.

By observ. true distance . .		77°	52′	45″
In Naut. Almanac dist. at . 3^h		77	14	40
6		78	47	24
Prop. logarithm	·67454 . .		38	5
	·28804 . .	1	32	44
	·38650 Cor.	1^h	11^m	55^s
	Adding	3		
Greenwich mean time Nov. 22		4	11	55

Find mean time at Greenwich from each of the following observations:—

(57.) November 24, 1853, when true distance of Aldebaran was 93° 38′ 45″. Ans., 3^h 57^m 18^s.

(58.) Sept. 24, 1853, when true distance of Regulus was 58° 45′ 8″. Ans., 16^h 3^m 6^s.

(59.) May 27, 1853, when true distance of the sun was 110° 8′ 50″. Ans., 14^h 2^m 22^s.

Elements from Nautical Almanac.

Dist. Aldebaran, Nov. 24, at	3^h .	93°	7′	57″	at 6^h.	94°	44′	52″
„ Regulus, Sept. 24 . „	15 .	59	16	16	„ 18 .	57	47	27
„ Sun, May 27 . . . „	mid. .	111	12	57	„ 15 .	109	38	38

To take out a planet's right ascension and declination.

Proceed as in the similar rules for finding the sun's right ascension and declination (pp. 74, 81).

The rules above given are sufficient to enable the student to acquire a knowledge of the principal contents of the Nautical Almanac. They will be continually referred to in the subsequent rules for finding the latitude and longitude.

We have supposed in the above examples the motion of the heavenly body to be *uniform* in the interval between the Greenwich times taken out of the Nautical Almanac. This is seldom the case, although in most of the questions in Nautical Astronomy it may be so assumed without any practical error. When, however, very accurate results are required, a correction must be used, called the *equation of second differences.* The investigation of this equation, and examples of its application, must be postponed for the present.

CHAPTER V.

PRELIMINARY PROBLEMS AND RULES IN NAUTICAL ASTRONOMY. CORRECTIONS FOR PARALLAX, REFRACTION, CONTRACTION OF THE MOON'S SEMIDIAMETER, AND DIP.

Given, mean solar time, and the equation of time, to find the apparent solar time; or,

Given, apparent solar time, and the equation of time, to find mean solar time.

Rule XVI.

1. Get a Greenwich date (p. 70.).
2. Correct the equation of time for this date (p. 77).
3. Apply the equation of time with its proper sign (as shown in the Nautical Almanac) to the given time.
4. The result is the time required.

EXAMPLES.

1. April 27, 1846, at $9^h\ 10^m$ P.M., mean time, in long. 16° W., required apparent solar time.

Ship, April 27	9^h	10^m
Long. in time	1	4 W.
Greenwich, April 27 .	10	14

			Equation of Time. (Page ii. Nautical Almanac.)
			27 . . . . 2^m $26^s{\cdot}9$ add to
			28 . . 2 36 ·4 m. time.
			9 ·5
Ship, April 27	9^h 10^m 0^s	m. time	Prop. logs. ·37020
Eq. of time .	2 31 ·0		3·05570
Ship, April 27	9 12 31 ·0	app. time required.	3·42590 4 ·1
			2 31 ·0 eq. of time.

2. June 22, 1852, at 5^h 42^m P.M., apparent solar time, in long. 100° 30′ E., required mean solar time.

Ship, June 22	5^h 42^m	P.M.
Long. in time	6 42	E.
Greenwich, June 21. .	23 0	

Equation of time.
(Page i. Nautical Almanac.)
Difference for 1 hour, $0^s{\cdot}54$.

21	1^m $27^s{\cdot}2$	add to
22	1 40 ·2	app. time.
	13 ·0	
23 × ·54 . .	12 ·4	
	1 39 ·6	

Ship, June 22 . .	5^h 42^m 0^s
Eq. of time . . .	1 39·6 +
∴ mean time . .	5 43 39·6

(60.) July 4, 1853, at 6^h 10^m P.M. mean time, in long. 100° W., required apparent solar time. Ans., 6^h 5^m $53^s{\cdot}2$.

(61.) Dec. 10, 1853, at 4^h 42^m P.M. apparent solar time, in long. 80° 45′ W., required mean solar time.

Ans., 4^h 35^m $18^s{\cdot}1$.

(62.) Feb. 23, 1848, at 10^h 40^m A.M. apparent solar time, in long. 1° 6′ W., required mean solar time.

Ans., 10^h 53^m $43^s{\cdot}5$.

Elements from Nautical Almanac.

Equation of time	July 4 .	4^m $1^s{\cdot}1$ sub.	July 5 .	4^m $11^s{\cdot}7$ sub.
„	Dec. 10 .	6 53 ·5 sub.	Dec. 11 .	6 25 ·8 sub.
„	Feb. 22 .	13 51 ·0 add.	Feb. 23 .	13 43 ·1 add.

Rule XVII.

Given, mean solar time, to find sidereal time.

1. Get a Greenwich date (p. 70).
2. Correct the right ascension of the mean sun by the table (p. 83), or by proportional logarithms, or otherwise for the Greenwich date.
3. Add together the corrected right ascension of mean sun and mean time at the ship.
4. The sum (rejecting 24 hours if greater than 24 hours) will be sidereal time.

EXAMPLES.

Feb. 24, 1848, at $10^h\ 40^m\ 30^s$ A.M. mean time, in long. 1° 6′ W., required sidereal time.

Ship, Feb. 23	22^h	40^m	30^s
Long. in time	0	4	24
Greenwich, Feb. 23 . .	22	44	54

	Right ascension mean sun.			
23	22^h	10^m	$3^s{\cdot}52$	
Cor. for 22^h . . .		3	36 ·84	by table.
„ 44^m . . .			7 ·23	
„ 54^s . . .			·15	
Right asc. mean sun	22	13	47 ·74	
Ship mean time . .	22	40	30 ·00	
Sidereal time . . .	20	54	17 ·74	
	(rejecting 24 hours.)			

(63.) July 10, 1853, at $0^h\ 42^m\ 10^s$ P.M. mean time, in long. 84° 42′ W., required sidereal time.

Ans., $7^h\ 56^m\ 29^s{\cdot}7$.

(64.) Sept. 30, 1853, at $6^h\ 42^m\ 10^s$ A.M. mean time, in long. 100° 42′ W., required sidereal time.

Ans., $7^h\ 18^m\ 58^s{\cdot}59$.

(65.) Dec. 8, 1853, at 10^h 10^m 42^s P.M., mean time, in long. 18° 32′ E., required sidereal time.

Ans., 3^h 20^m 47^s.

Elements from Nautical Almanac.

Right ascen. mean sun, July 10	.	7^h	13^m	$17^s{\cdot}14$
„ Sept. 30	.	12	36	34 ·64
„ Dec. 8	.	17	8	36 ·96

Rule XVIII.

Given, apparent solar time, to find sidereal time.

1. Get a Greenwich date (p. 70.)
2. Correct the equation of time and also the right ascension of the mean sun for Greenwich date (pp. 77, 83).
3. Apply corrected equation of time to ship apparent time, and thus get ship mean time. Then, as in the last rule,
4. Add together ship mean time and right ascension of mean sun.
5. The sum (rejecting 24 hours if greater than 24 hours) will be sidereal time.

EXAMPLES.

May 24, 1853, at 6^h 8^m 40^s A.M. apparent solar time, in long. 20° 20′ W., required sidereal time.

Ship, May 23 . . .	18^h	8^m	40^s	
Long. in time . . .	1	21	20	W.
Greenwich, May 23 .	19	30	0	

Equation of time.

23	3^m	$33^s{\cdot}2$	sub. from
24	3	28 ·3	app. time.
		4 ·9	

Prop. logs.

·09018			
3·34323			
3·43341		4·0	
Eq. of time		8	29·2
S. app. T.	18	8	40·0
May 23 .	18	5	18·8 Ship mean time

Right ascension mean sun.

23rd, at noon . . .	4^h	4^m	$2^s{\cdot}37$
19^h		3	7 ·27
30^m			4 ·93
R. A. mean sun . .	4	7	14 ·57
Ship M. T.	18	5	18 ·80
Sidereal time. . .	22	12	25 ·37

(66.) July 4, 1853, at $3^h\ 42^m$ A.M. apparent solar time, in long. 84° 42′ W., required sidereal time.

Ans., $22^h\ 35^m\ 11^s{\cdot}53$.

(67.) Oct. 21, 1853, at $8^h\ 48^m$ P.M. apparent solar time, in long. 88° 8′ E., required sidereal time.

Ans., $22^h\ 32^m\ 30^s{\cdot}87$.

Elements from Nautical Almanac.

Equation of time.						Right ascen. mean sun.
July 3,	$3^m\ 50^s{\cdot}1$ add	4,	$4^m\ 1^s{\cdot}1$, add	. . on	3,	$6^h\ 45^m\ 41^s{\cdot}24$
Oct. 21,	15 19·1 sub.	22,	15 28·1, sub.	. . „	21,	13 59 22·26

Rule XIX.

Given, mean time, or apparent time at the ship, to find what heavenly body will pass the meridian the next after that time.

1. Get a Greenwich date (p. 70).

2. Find the right ascension of the mean sun (and, if the Greenwich date is in apparent time, find also the equation of time, p. 77) for that date, so as to get mean time (p. 88).

3. Add together ship mean time and the right ascension of mean sun.

4. The sum (rejecting 24 hours if greater than 24 hours) will be sidereal time, or the right ascension of the meridian.

5. Then that star found in some catalogue of fixed stars, whose right ascension is the next greater, will be the star required.

EXAMPLE.

Feb. 24, 1853, at $4^h\ 42^m$ P.M. mean time nearly, in long. 100° E., find what bright star will pass the meridian the next after that date.

Ship, Feb. 24	4^h	42^m
Long. in time . . .	6	40 E.
Greenwich, Feb. 23 . .	22	2

Right ascension mean sun.								
23	22^h	13^m	$9^s{\cdot}0$	Ship, Feb. 24 . . .	4^h	42^m	0^s
22^h . .		3	36·8	Right asc. mean sun	22	16	46
2^m . .			·3	Right asc. of merid.	2	58	46
	22	16	46·1				

Looking into the "Catalogue of the mean places of 100 principal fixed stars" (Nautical Almanac, p. 432), we find the star whose right ascension is next greater than $2^h\,58^m$ is *α* Persei; therefore *α* Persei is the bright star that will come to the meridian the next after $4^h\,42^m$ P.M. on Feb. 24.

Sometimes it is required to find what principal stars will pass the meridian between certain convenient hours for observing their transits: as, for instance, between 8^h and 11^h P.M. To do this, we must find the right ascension of the meridian for these two times by the above rule; then the stars whose right ascensions lie between will be the stars required.

EXAMPLES.

Oct. 3, 1853, in long. 90° W., find what bright stars put down in the Nautical Almanac will pass the meridian between the hours of 9 and 12 P.M.

Ship, Oct. 3	9^h	0^m		Ship, Oct. 3	12^h	0^m	
Long. in time	6	0 W.		Long. in time . . .	6	0 W.	
Greenwich, Oct. 3 . .	15	0		Greenwich, Oct. 3 . .	18	0	

Right ascension mean sun.				Right ascension mean sun.			
Oct. 3	12^h	48^m	24^s	Oct. 3	12^h	48^m	24^s
15^h		2	27	18^h		2	57
	12	50	51		12	51	21
Ship, Oct. 3 . . .	9	0	0	Ship, Oct. 3 . . .	12	0	0
Right asc. meridian .	21	50	51	Right asc. meridian	0	51	21

In Catalogue p. 432, Nautical Almanac, the stars whose right ascensions lie between $21^h\,50^m\,51^s$ and $0^h\,51^m\,21^s$ are from *α* Aquarii to *β* Ceti inclusive.*

* In the "Handbook for the Stars," published by the author, there is a table of the times of the transits of the principal fixed stars. This

(68.) What bright stars put down in the Nautical Almanac will pass the meridian of a ship in long. 40° E., between 8^h and 10^h P.M. mean time on Nov. 20, 1853?

Ans., From α Andromedæ to α Arietis.

(69.) What bright star will pass the meridian of a ship in long. 30° W. the first after 10^h 30^m P.M. on Oct. 10, 1853?

Ans., α Andromedæ.

(70.) What bright stars will pass the meridian of a ship in long. 56° W. between the hours of 6 and 10 P.M. on March 10, 1853? Ans., From β Tauri to γ Argûs.

(71.) What bright stars put down in the Nautical Almanac will pass the meridian of Greenwich between the hours of 7 and 9 P.M. mean time, on August 20, 1853?

Ans., From ϵ Ursæ Minoris to β Lyræ.

(72.) What stars named in the Nautical Almanac will pass the meridian of a ship in long. 86° E., on Oct. 20, 1853, between the hours of 10 P.M. and midnight?

Ans., From α Andromedæ to α Eridani.

(73.) What bright star will pass the meridian of Greenwich the first after 9^h P.M. on Sept. 12, 1853?

Ans., α Cygni.

Elements from Nautical Almanac.

	Right ascension mean sun.		
November 20 . . .	15^h	57^m	39^s
March 10	23	12	17
October 10. . . .	13	16	0
August 20	9	54	56
October 20. . . .	13	55	26
September 12 . . .	11	25	37

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table enables the observer to find the name of the bright star that is on the meridian at any given time, and at any place, without calculation.

Rule XX.

Given, sidereal time, to find mean time.

1. Take out of the Nautical Almanac the right ascension of the mean sun (called in the Nautical Almanac sidereal time), for noon of the given day.
2. From sidereal time (increased if necessary by 24 hours) subtract the quantity just taken out, the remainder is mean time nearly.
3. Find, in the table of the acceleration of sidereal on mean solar time, the correction for this time, and subtract it from the mean time nearly.
4. The remainder is the mean time required.

NOTE.—In strictness we ought to have entered the table with the correct mean time, instead of that used; but it is evident we may obtain a still closer approximation to the truth by repeating the work, using the last approximate value instead of the preceding one. For all practical purposes this repetition is unnecessary.

EXAMPLES.

1. April 27, 1846, when a sidereal clock showed $3^h\ 40^m\ 45^s$, required mean time.

Sidereal time				3^h	40^m	$45^s{\cdot}0$
Right as. mean sun at mean noon				2	20	21·58
Mean time nearly				1	20	23·42
Cor. . . .	1^h . .	$9^s{\cdot}86$				
Naut. Alm.	20^m .	3·28				
p. 592.	23^s .	·06				
		13·20 .				13·20
Mean time required				1	20	10·22

2. March 2, 1848, when a sidereal clock showed $3^h\ 40^m\ 45^s$, required mean time.

Sidereal time		3^h	40^m	$45^s{\cdot}0$
Right asc. m. sun at m. noon		22	41	35 ·94
Mean time nearly		4	59	9 ·06
Cor. 4^h . .	$39^s{\cdot}43$			
59^m . .	9 ·69			
9^s . .	·02			
	49 ·14			49 ·14
Mean time required . . .		4	58	19 ·92

The clock of an observatory used for taking the transits of a heavenly body is generally adjusted to sidereal time: the above rule will show how to determine the error of a solar clock, or a chronometer regulated to mean time; for we have only to note the time shown by the two instruments at the same instant by comparing the chronometer with the sidereal clock at some coincident beat, the error of the latter being supposed to be known.

EXAMPLE.

Greenwich, March 3, 1853, when a sidereal clock showed $6^h\ 10^m\ 20^s$ a chronometer showed $7^h\ 32^m\ 10^s$, required the error of the chronometer on Greenwich mean time; the error of sidereal clock being $2^m\ 42^s{\cdot}5$ slow.

Sidereal clock		6^h	10^m	20^s
Error			2	42·5 slow
Sidereal time		6	13	2·5
Right asc. mean sun at mean noon		22	44	41·48
Cor. 7^h . .	$1^m\ 8^s{\cdot}99$	7	28	21·02
28^m . .	4 ·60			
21^s . .	·05			
	1 13 ·64		1	13·64
Required mean time		7	27	7·38
Chronometer showed		7	32	10·0
Error of chro. on Gr. mean time			5	2·62 fast

When the calculations are made for any other meridian than that of Greenwich, for which the quantities in the Nautical Almanac are calculated, we must take into consideration the change of the mean sun's place arising from the difference of longitude. For example, the tables of the *Connaissance des Tems* are computed for Paris, the long. of which is $9^m\ 22^s$ to the east of Greenwich: as in that time the mean sun moves to the eastward through an arc of $1^s{\cdot}53$ in time (for $24^h : 9^m\ 22^s :: 3^m\ 56^s{\cdot}55 : 1^s{\cdot}53$), it follows that we must add $1^s{\cdot}53$ to all the right ascensions of the mean sun in the French tables to obtain those of the mean sun at mean noon at Greenwich.

EXAMPLE.

April 27, 1841, the right ascension of the mean sun at mean noon at Paris, by the *Connaissance des Tems*, was $2^h\ 21^m\ 10^s{\cdot}09$, required the same for Greenwich mean noon.

Greenwich, April 27 . .		0^h	0^m	0^s
Long. in time			9	22 W.
Paris date April 27 . . .			9	22
Right asc. mean sun at Paris . .		2^h	21^m	$10^s{\cdot}09$
Cor. 9^m .	$1^s{\cdot}48$			
22^s .	$\cdot 05$			
	1 ·53			1 ·53
Right asc. mean sun at Greenwich		2	21	11 ·62

The longitude is usually found at sea by means of a chronometer showing Greenwich mean time at the instant the mean time at the ship is known. The mean time at the ship is deduced from the hour angle of a heavenly body, and this hour angle is calculated by means of the altitude of the body observed with a sextant and certain elements found in the Nautical Almanac.

Rules for finding the hour angle of a heavenly body will be given hereafter. We will therefore suppose the hour angle known, and proceed to show how mean time might be found from it.

Rule XXI.

Given, the hour angle of a heavenly body, to find mean time at the ship.

*Hour angle found by table of haversines.**

When the heavenly body is *west* of meridian, take the hour angle out at top of page of haversines and add thereto the right ascension of the heavenly body: from the sum (increased if necessary by 24 hours) subtract the right ascension of mean sun (corrected for estimated mean time at Greenwich or Greenwich date), the result (rejecting 24 hours if greater than 24 hours) is the mean time required.

When the heavenly body is *east* of meridian, take the hour angle out of the table at *bottom* of page of haversines, then proceed as directed above.

If the heavenly body observed is the sun, the hour angle taken out will also be *apparent* time (p. 64, art. 24); the mean time will then be found by applying the equation of time with the proper sign as shown in the Nautical Almanac.

Hour angle found by any other table.

The angle taken out turned into time (if necessary) will also be the hour angle if the body be west of meridian; but if the body be east of meridian, subtract the hour angle from 24 hours, and then, as in the former case, add thereto the star's right ascension, and from the sum subtract the right ascension of mean sun as pointed out in the rule.

* 24^h — Star's hour angle can be taken out of Inman's table of haversines *by inspection:* when, therefore, the longitude by chronometer is found by means of this table, the above rule applies whether the heavenly body is east or west of meridian.

EXAMPLE.

August 11, 1846, at 8^h 50^m P.M., mean time nearly, in long. 90° W., the hour angle of Arcturus was 3^h 56^m 55^s west of meridian, required correct mean time at the place.

Ship, Aug. 11 . . .	8^h	50^m
Long. in time . . .	6	0 W.
Greenwich, Aug. 11 .	14	50

Right ascension mean sun.			
Aug. 11	9^h	18^m	$16^s{\cdot}51$
Cor. 14^h . . .		2	17 ·99
50^m . . .			8 ·21
.	9	20	42 ·71

Star's hour angle.	3^h	56^m	$55^s{\cdot}$ 0
Star's right asc. .	14	8	40 ·14
	18	5	35 ·14
Rt. asc. mean sun	9	20	42 ·71
Ship mean time .	8	44	52 ·43

This result is slightly incorrect, arising from the estimated mean time, 8^h 50^m, being different from the true time. When great accuracy is required, the operation should be repeated, using mean time last found, namely 8^h 45^m, instead of the one used before; thus,

The operation repeated.

Ship, Aug. 11	8^h	45^m
Long. in time	6	0
Greenwich, Aug. 11 . .	14	45

Right ascension mean sun.			
Aug. 11	9^h	18^m	$16^s{\cdot}51$
14^h . . .		2	17 ·99
45^m . . .			7 ·39
	9	20	41 ·89

Star's hour angle .	3^h	56^m	$55^s{\cdot}0$
Star's right asc. .	14	8	40 ·12
	18	5	35 ·12
Rt. asc. mean sun	9	20	41 ·89
Cor. ship mean t.	8	44	53 ·23

(74.) Nov. 22, 1853, at 7^h 15^m P.M., mean time nearly, in long. 22° 0′ W., the hour angle of Aldebaran (α Tauri) was 5^h 10^m 20^s east of meridian, required mean time at the place.

Ans., 17^h 30^m 54^s.

(75.) June 23, 1853, at 3^h 15^m A.M., mean time nearly, in

long. 100° 40′ E., the hour angle of α Lyræ was $3^h\ 42^m\ 40^s$ west of meridian, required mean time at the place.

Ans., $16^h\ 11^m\ 6^s$.

(76.) June 15, 1853, at $10^h\ 10^m$ P.M., supposed mean time nearly, in long. 10° 42′ W., the hour angle of Arcturus was $2^h\ 2^m\ 30^s$ east of meridian, required mean time at the place.

Ans., $6^h\ 30^m$ first approximation;
$6^h\ 30^m\ 35^s$ more exactly.

Elements from Nautical Almanac.

Right ascension mean sun.				Right ascension star.			
Nov. 22, 1853	16^h	5^m	32^s	Aldebaran	4^h	27^m	32^s
June 22, „	6	2	10	α Lyræ	18	32	0
June 15, „	5	34	43	Arcturus	14	8	59

Rule XXII.

To find at what time any heavenly body will pass the meridian.

1. Take out of the Nautical Almanac the right ascension of the given star, and also the right ascension of the mean sun for noon of the given day.

2. From the right ascension of the star (increased if necessary by 24 hours) subtract the right ascension of the mean sun, the remainder is mean time at the ship nearly.

3. Apply the longitude in time, and thus get a Greenwich date; with this Greenwich date correct the right ascension of mean sun for Greenwich date.

4. Then from the right ascension of the star subtract the right ascension of the mean sun thus corrected, the remainder is the mean time when the heavenly body is on the meridian.

As in the last problem, the table of acceleration ought to have been entered with the correct mean time; but the error for all practical purposes is inappreciable.

EXAMPLE.

At what time will Sirius pass the meridian of a place in long. 68° 30′ W. on Nov. 20, 1845?

Star's right ascension . .	6^h	38^m	23^s	$+ 24^h$
Right asc. m. sun at noon .	15	57	26	
Ship, Nov. 20	14	40	57	M. T. nearly
Long. in time	4	34	0	
Greenwich, Nov. 20 . . .	19	15	57	

Right ascension mean sun.

Nov. 20	15^h	57^m	26^s	Star's rt. asc. + 24^h .	30^h	38^m	23^s
19^h		3	7	Rt. asc. mean sun .	16	0	35
16^m			2	Ship Nov. 20 . . .	14	37	48
Rt. asc. mean sun .	16	0	35				

Therefore the transit of Sirius is at $14^h\ 37^m\ 48^s$ on Nov. 20, or at $2^h\ 37^m\ 48^s$ A.M. on Nov. 21.

To find at what time it will pass the meridian on the morning of Nov. 20, we must evidently begin one day back and take out the right ascension of the mean sun for Nov. 19.

(77.) At what time will α Pegasi pass the meridian of Portsmouth, long. 1° 6′ W., on Nov. 25, 1853?

Ans., Nov. 25, $6^h\ 38^m\ 58^s$.

(78.) At what time will the star Regulus (α Leonis) pass the meridian of Land's End, long. 5° 42′ W., on May 30, 1845? Ans., May 30, $5^h\ 27^m\ 45^s$ P.M.

(79.) At what time will Antares pass the meridian of Portsmouth, long. 1° 6′ W., on Aug. 20, 1845?

Ans., Aug. 20, $6^h\ 24^m\ 11^s$.

(80.) At what time will α Leonis pass the meridian of Lisbon, long. 9° 8′ W., on June 4, 1846?

Ans., June 4, $5^h\ 9^m\ 4^s$.

(81.) At what time will the star Antares pass the meridian of Copenhagen, long. 12° 35′ E., on Aug. 20, 1846?

Ans., Aug. 20, $6^h\ 25^m\ 21^s$.

(82.) At what time will the star Fomalhaut pass the meridian of Calcutta, long. 88° 26′ E., on Nov. 20, 1846?

Ans., Nov. 20, $6^h\ 52^m\ 34^s$.

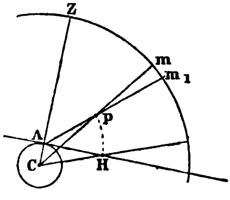
Elements from Nautical Almanac.

	Right ascension mean sun.			Right asc. of star.		
Nov. 25, 1853	16^h	17^m	22^s	22^h	57^m	26^s
May 30, 1845	4	31	25	10	0	9
Aug. 20, „	9	54	43	16	19	58
June 4, 1846	4	50	11	10	0	11
Aug. 20, „	9	53	45	16	20	2
Nov. 20, „	15	56	28	22	49	11

We will conclude this chapter by giving brief explanations of some of the principal corrections required for reducing the observations used for finding the latitude, longitude, time at the ship, and variation of the compass—the subjects of the next chapter.

Correction for parallax.

The place of a heavenly body as seen, or supposed to be seen, from the centre of the earth, is called its *true*, or *geocentric* place: the place of a heavenly body as seen from any point on the earth's surface is called its *apparent* place.



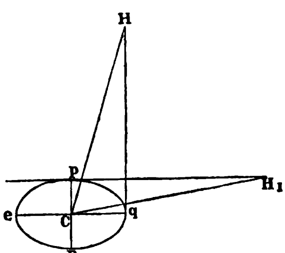
Thus, let A be the place of a spectator on the surface of the earth, p any heavenly body, as the moon. Through p draw the straight lines A p $m_{,}$, C p m from the surface and centre to the celestial concave; then m is the true place, and $m_{,}$ the apparent place of the heavenly body p. The arc m $m_{,}$, or angle A p C, is called the diurnal parallax.

It appears from the figure, that the effect of parallax is to *depress* bodies in a plane passing through the reduced zenith, which coincides nearly with a vertical plane; the diurnal

parallax A *p* C is therefore usually called the *parallax in altitude.* If H be the heavenly body in the horizon of the spectator, the angle A H C is called the *horizontal parallax* of *p*.

It is also evident from the figure that the parallax of a heavenly body is greatest when in the horizon, and that it diminishes to zero in the reduced zenith; that the parallax for different bodies will differ, depending on their distance from the spectator: that the nearer the body is to the earth the greater will be its parallax: thus the moon's parallax is the greatest of any of the heavenly bodies: the fixed stars, with perhaps a few exceptions, are at such an immense distance, that the earth dwindles to a point so indefinitely small that the line A C subtends no measurable angle at a star: hence the fixed stars are considered to have no parallax.

Since the form of the earth is considered to be an oblate spheroid, the equatorial diameter being about 26 miles longer than the polar diameter or axis, the horizontal parallax of a heavenly body, as observed from some place on the equator, will be greater than the horizontal parallax of the same heavenly body if observed from the poles of the earth. For let *q* be a spectator at the equator, and H a heavenly body in his horizon, then the angle H is the equatorial horizontal parallax of the body at H. Similarly to a spectator at *p* the pole of the earth, the horizontal parallax of the same body would be $H_{,,}$

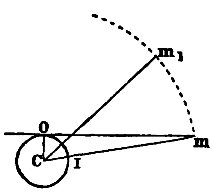


which is evidently less than H, since it is subtended by a smaller radius of the earth; thus it appears from the figure that the horizontal parallax is greatest at the equator, and that it diminishes as the latitude increases. The moon's horizontal parallax put down in the Nautical Almanac is the *equatorial* horizontal parallax. To find the horizontal

parallax for any other place a correction must be applied, which is evidently subtractive: this correction is seldom made in the common problems of navigation: in finding the longitude by occultations or solar eclipses, it ought not to be omitted. It is inserted in most collections of Nautical Tables.

Correction or augmentation of the moon's horizontal semidiameter.

The moon's semidiameter given in the Nautical Almanac is the *horizontal* semidiameter. When the moon is above the horizon its diameter appears under a greater angle, since



the body has approached nearer the earth; for the distance of the moon at *m* from the centre of the earth being a little more than sixty times the radius of the earth, C $m = 60 \times$ C I. As the horizontal parallax, C *m* O, is about 1° only, the line *m* O is nearly equal to *m* C. Hence two observers placed, the one at O, the other at I, would see the moon, the first in his horizon, the other in his zenith: but O would see the heavenly body distant a little more, and I a little less, than sixty times the radius; the diameter in fact would appear to the former about 30″ less than to the latter. At any intermediate point as at $m_{,}$ the moon's semidiameter would evidently appear to be greater than at O, and less than at I. The correction to be made to the moon's horizontal semidiameter on this account is called the *augmentation.* It has been computed for every degree of altitude, and may be found in the Nautical Tables.

Correction for refraction.

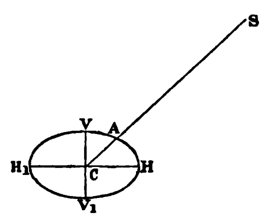
A ray of light passing obliquely from one medium to another of greater density, is found to deviate from its rectilineal course, and to bend towards a perpendicular to

the surface of the denser medium. Hence to a spectator on the earth's surface, a heavenly body seen through the atmosphere appears to be raised, and its true place, on this account, is below its apparent place. Observations show that refraction is greatest when the body is in the horizon (about 34′), and that it diminishes to zero in the zenith. A table of refractions for every altitude has been formed and inserted in the Nautical Tables.

The corrections for parallax and refraction are frequently combined, so that they form one correction, called the "correction in altitude." The two tables of the correction in altitude for the sun and moon may also be found in most collections of nautical tables.

Correction for the contraction of the moon's semidiameter on account of refraction.

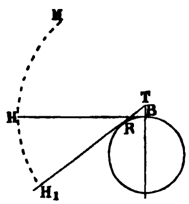
When the moon is near the horizon its disc assumes an elliptical form resembling H_1 A H, in consequence of the unequal effect of refraction at low altitudes, the lower limb being raised more than the centre, and the centre more than the upper limb. If, therefore, in a lunar observation a contact is made between a distant object S and some point A on the moon's limb, the contracted semidiameter C A must be added to the arc A S to obtain the distance S C of the centres, and not C H, the moon's uncontracted semidiameter, which is evidently too great. This correction has been calculated, and may be found in the Nautical Tables.



Correction for dip.

The altitude of a heavenly body, observed from a place above the surface of the earth, as on the deck of a ship,

will evidently be greater than its altitude observed from the surface, since the observer brings the image of the body down to his horizon, which is lower than the horizon seen from the surface of the sea immediately below him. The difference of altitude from this cause expressed in minutes and seconds, is called the *dip* of the sea horizon. Let a



tangent at B, the point on the surface beneath the spectator supposed to be at T, meet the celestial concave at H, and through T draw the tangent T H, touching the earth at R; then, if M be the place of a heavenly body, the arc M H is its altitude observed at B, and M $H_{\prime}$ the altitude observed by the spectator at T: the arc H $H_{\prime}$ is the dip due to the height B T of the spectator above the surface of the sea, and is evidently subtractive, to get the true altitude. This correction is found in all collections of nautical tables.

The use of the preceding corrections and reductions will be best seen in the following examples.

Rule XXIII.

Given, a star's observed altitude, to find its true altitude.

The stars are such a distance from the spectator that (excepting probably a few) the earth's orbit subtends no angle at the star: hence a star is considered to have no parallax: and the only corrections used for reducing the observed altitude to the true are the index correction (the correction of the quadrant or sextant used) the dip, and refraction. Hence this rule.

1. To the observed altitude apply the index correction with its proper sign.
2. Subtract the dip (taken from table of dip of horizon).
3. Subtract the refraction (taken from table of refraction).
4. The result is the true altitude of star.

EXAMPLE.

The observed altitude of Arcturus was 36° 10′ 20″, index correction + 2′ 42″, and height of eye above the sea was 20 feet, required the true altitude.

Observed altitude . . .	36° 10′ 20″
Index correction . . .	2 42 +
	36 13 2
Dip	4 24 —
Star's apparent altitude .	36 8 38
Refraction	1 20 —
Star's true altitude . .	36 7 18

(83.) The observed altitude of Aldebaran was 13° 4′ 30″, index correction — 10′ 40″, and height of eye above the sea was 16 feet; required the true altitude.

Ans., 12° 45′ 43″.

(84.) The observed altitude of γ Tauri was 62° 42′ 15″, index correction + 0′ 40″, and height of eye above the sea was 20 feet: required the true altitude.

Ans., 62° 38′ 1″.

(85.) The observed altitude of α Canis Majoris (Sirius) was 32° 42′ 30″, index correction was — 3′ 30″, and height of eye above the sea was 12 feet: required true altitude.

Ans., 32° 34′ 0″.

Rule XXIV.

Given, a planet's observed altitude, to find its true altitude.

The effect of parallax on the true altitude of a heavenly body is to diminish it (p. 102): the correction of parallax in altitude must therefore be added to the observed, to get the true altitude. Hence this rule.

Correct the observed altitude for index correction, dip, and refraction, as in (1), (2), (3), p. 106.

(4.) To the result add the parallax in altitude (taken out of the table of parallax in altitude of sun and planets).

(5.) The result is the true altitude of the planet.

EXAMPLE.

January 4th, 1848, the observed altitude of Mars was 21° 41′ 10″, index correction 24 feet: horizontal parallax in Nautical Almanac being 10·1: required the true altitude.

Observed altitude . . .	21°	41′	10″	
Index correction . . .		2	42	+
	21	43	52	
Dip		4	49	—
	21	39	3	
Refraction		2	27	—
	21	36	36	
Parallax in altitude . .			9	+
True altitude	21	36	45	

(86.) Jan. 24, 1848, the observed altitude of Mars was 9° 8′ 30″, index correction — 3′ 45″, and height of eye above the sea 16 feet: required the true altitude. The horizontal parallax from Nautical Almanac was 8″·3.

Ans., 8° 55′ 3″.

(87.) Feb. 3, 1848, the observed altitude of Venus was 25° 8′ 30″, index correction — 10′ 50″, and height of eye above the sea 12 feet, required the true altitude. The horizontal parallax from Nautical Almanac, was 8″·1.

Ans., 24° 52′ 17″.

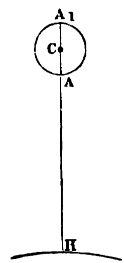
(88.) Jan. 30, 1848, the observed altitude of Jupiter was 10° 20′ 10″, the index correction was + 0′ 14″, and height of eye above the sea 18 feet: required the true altitude, the horizontal parallax in Nautical Almanac being 2″·0.

Ans., 10° 11′ 0″.

Rule XXV.

Given, the sun's observed altitude, to find the true altitude.

The true altitude of the sun's centre C H is found by observing the altitude of either the upper or lower limb A′ H or A H, and then subtracting or adding the semidiameter C A taken from the Nautical Almanac; the other corrections, namely, for the instrument, dip, refraction, and parallax, being made as in the preceding rules. In some of the nautical tables, the two corrections for refraction and parallax of the sun are combined in one table, and called the "correction in altitude of the sun."



Hence this rule.

1. Correct the observed altitude for index correction and dip, as in article (1), (2), p. 106.

2. To this add the sun's semidiameter, if the altitude of the lower limb is observed; but subtract if the upper limb is observed: the result is the apparent altitude of the sun's centre.

3. Subtract the refraction and add the parallax taken from the proper tables: or rather take out the "correction in altitude of the sun," and subtract it.

4. The remainder is the sun's true altitude.

EXAMPLE.

The observed altitude of the sun's lower limb (L. L.) was 47° 32′ 15″, the index correction was + 2′ 10″, and the height of the eye above the sea 15 feet: required the true altitude of the sun's centre: the semidiameter in Nautical Almanac being 15′ 49″.

Observed altitude . .	47°	32′	15″	
Index correction . .		2	10	+
	47	34	25	
Dip		3	49	—
	47	30	36	
Semidiameter . . .		15	49	+
Apparent altitude . .	47	46	25	
Correction in altitude			47	—
True altitude . . .	47	45	38	

(89.) The observed altitude of the sun's L. L. was 48° 30′ 15″, index correction — 2′ 50″, and height of eye above the sea 15 feet: required the true altitude, the semidiameter being 15′ 55″. Ans., 48° 38′ 46″.

(90.) The observed altitude of the sun's L. L. was 40° 42′ 16″, index correction + 5′ 10″, and height of eye above the sea 20 feet: required the true altitude, the semidiameter being 16′ 4″. Ans., 40° 58′ 6″.

(91.) The observed altitude of the sun's upper limb (U. L.) was 55° 57′ 42″, index correction — 3′ 40″, height of eye above the sea 19 feet: required the true altitude, the semidiameter being 16′ 6″. Ans., 55° 33′ 4″.

(92.) The observed altitude of the sun's L. L. was 39° 25′ 15″, index correction — 3′ 15″, height of eye above the sea was 15 feet: required the true altitude, the semidiameter being 16′ 3″. Ans., 39° 33′ 11″.

Rule XXVI.

Given, the moon's observed altitude, to find the true altitude.

The moon's horizontal parallax and semidiameter change so perceptibly that they cannot be considered (as in the corresponding case of the sun) to be constant for 24 hours. The parallax and semidiameter taken out of the Nautical Almanac must therefore be corrected for the Greenwich date in order to find the horizontal parallax and horizontal semidiameter at the time of the observation. Moreover

since the moon is nearer the earth when observed than when it was in the horizon, the horizontal semidiameter must also be corrected for augmentation (p. 104). The correction of the moon's apparent altitude for parallax and refraction is found inserted in most of the nautical tables: it is entered with the corrected horizontal parallax at top, and the apparent altitude at the side. Hence this rule.

1. Get a Greenwich date.
2. Correct the moon's semidiameter and horizontal parallax, taken from the Nautical Almanac, for the Greenwich date (p. 78).
3. Add to the semidiameter the augmentation, taken from the table of augmentation.
4. Correct the observed altitude for index correction, dip, and semidiameter, as in the preceding rules (p. 106).
5. Add the moon's correction in altitude, taken out of the proper table.
6. The result is the moon's true altitude.

EXAMPLE.

April 7, 1853, at 4^h 47^m P.M., mean time nearly, in long. 10° W., the observed altitude of the moon's L. L. was 72° 15′ 0″, the index correction was — 4′ 20″, and height of eye above the sea 15 feet: required the true altitude.

Ship, April 7 . . .	4^h	47^m
Long. in time . .	0	40 W.
Greenwich, April 7 .	5	27

Moon's semidiameter.			Moon's horizontal parallax.		
7th noon	15′	40″·7	Noon	57′	32″·0
mid.	15	45 ·8	Mid.	57	50 ·8
		5 ·1			18 ·8
·34279			·34279		
3·32585			2·75927		
3·66864 cor. . . .		2 ·3	3·10206 cor. . . .		8 ·5
	15	43 ·0	Cor. hor. par. . . .	57	40 ·5
Aug.		15 ·2			
Aug. semi.	15	58 ·2			

Observed altitude	72°	15′	0″	
Index correction		4	20	—
	72	10	40	
Dip		3	49	—
	72	6	51	
Semidiameter		15	58	
Apparent altitude	72	22	49	
Correction in altitude . . {		16	58	
			12	
True altitude	72	39	59	

(93.) July 12, 1848, at 9^h 18^m P.M., mean time nearly, in long. 44° 40′ W., the observed altitude of the moon's L. L. was 27° 56′ 40″, the index correction + 2′ 20″, and height of eye above the sea 20 feet, required the true altitude.

Ans., 28° 56′ 9″.

(94.) May 15, 1848, at 10^h 25^m P.M., mean time nearly, in long. 55° 40′ W., the observed altitude of the moon's L. L. was 21° 14′ 10″, the index correction + 2′ 20″, and height of eye above the sea 15 feet, required the true altitude.

Ans., 22° 15′ 17″.

(95.) May 15, 1848, at 10^h 22^m P.M., mean time nearly, in long. 41° 30′ W., the observed altitude of the moon's U. L. was 45° 20′ 30″, the index correction + 4′ 10″, and height of eye above the sea 20 feet, required the true altitude.

Ans. 45° 42′ 34″.

Elements from Nautical Almanac.

	Moon's semidiameter.		Moon's horizontal parallax.
July 12, mid. . .	14′ 55″·9	. . . . mid. . .	54′ 47″·8
13, noon . .	14 59 ·3	. . . . noon . .	55 0 ·3
May 15, mid. . .	14 41 ·1	. . . . mid. . .	53 57 ·0
16, noon . .	14 42 ·3	. . . . noon . .	53 57 ·7

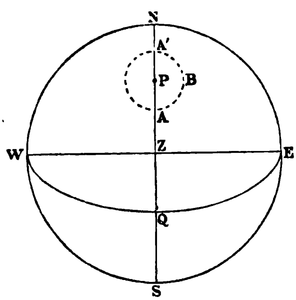
SECTION II.

RULES FOR FINDING THE LATITUDE, LONGITUDE, ERROR AND RATE OF CHRONOMETERS, AND VARIATION OF THE COMPASS.

CHAPTER VI.

RULES FOR FINDING THE LATITUDE.

To find the latitude by the meridian altitudes of a heavenly body above and below the pole.



Let N W S E represent the horizon of the spectator, Z the zenith, N Z S the celestial meridian, N P the altitude of the pole, W Q E the celestial equator. Then N P (the altitude of the pole) = latitude of spectator.* Let A B A' be a parallel of declination described by a heavenly body about the pole P,

then A' N = meridian altitude below pole
A N = " above pole
and A P = A' P = star's polar distance
∴ A' N = P N − A' P = lat. − star's polar distance
and A N = P N + A P = lat. + star's polar distance
adding A' N + A N = 2 lat.
or lat. = $\frac{1}{2}$ (A' N + A N) = half sum of latitudes.

* For ZN = PQ (each being 90°)
or, PN + PZ = PZ + ZQ
∴ PN = ZQ = latitude of spectator. See "Problems in Astronomy," by the Author.

If the heavenly body when passing the meridian above and below the pole, is on different sides of the zenith, so that the altitudes are taken from opposite sides of the horizon, subtract the greater altitude from 180°, so as to reduce it to an altitude taken from the same point of the horizon as the other altitude (see Exercise 2, p. 115).

Hence this

Rule XXVII.

To find the latitude by the meridian altitudes of a circumpolar star.

1. Correct the altitudes for index correction, height of eye, refraction and parallax (or as many of these as are applicable to the case), and thus get the true meridian altitudes.

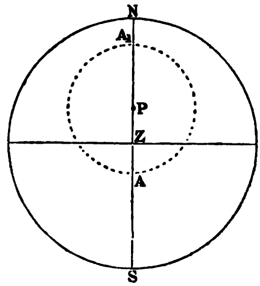
2. Add together the true meridian altitudes (reckoning from the same point of horizon), and half the result will be the latitude of the spectator.

EXAMPLES.

1. The meridian altitudes of α Ursæ Majoris were observed above and below the north pole to be 74° 10′ 10″ and 32° 42′ 15″ respectively (zenith south at both observations), index correction — 2′ 10″, and height of eye above the sea 20 feet, required the latitude.

Obs. alt. . . .	74° 10′ 10″		
Index cor. . .	2 10 —	Obs. alt.	32° 42′ 15″
	74 8 0	Index cor. . . .	2 10 —
Dip	4 24 —		32 40 5
	74 3 36	Dip	4 24 —
Refr.	17		32 35 41
1st true alt. .	74 3 19	Refr.	1 31
2nd „ „ . .	32 34 10	True alt. . . .	32 34 10
2)	106 37 29		
Latitude . . .	53 18 44·5 N.		

2. The meridian altitudes of α Aurigæ (Capella), were observed above and below the north pole to be 81° 10′ 52″ (zenith north of star), and 3° 42′ 52″ (zenith south), index correction — 3′ 10″, and height of eye above the sea 14 feet, required the latitude.



Obs. alt. .	3°	42′	52″
Index cor..		3	10—
	3	39	42
Dip . . .		3	41—
	3	36	1
		12	56—
True alt. .	3	23	5 (a)

(a) reckoned from N. point of hor.

Observed altitude . .	81°	10′	52″	
Index correction . .		3	10	—
	81	7	42	
Dip		3	41	—
	81	4	1	
			9	—
True altitude, A.S. .	81	3	52	
	180	0	0	
True altitude, A.N. .	98	56	8	(a)
	3	23	5	
	2)102	19	13	
Latitude	51	9	46·5	N.

(96.) The meridian altitudes of a star were observed above and below the north pole to be 69° 20′ 45″ and 6° 14′ 30″ respectively (zenith south at both observations), index correction — 1′ 45″, and height of eye 16 feet, required the latitude. Ans., lat. 37° 37′ 35″ N.

(97.) The meridian altitudes of a star were observed above and below the north pole to be 85° 10′ 10″ and 10° 10′ 10″ respectively (zenith south at both observations), index correction, — 2′ 40″, and height of eye 20 feet, required the latitude. Ans., lat. 47° 30′ 24″ N.

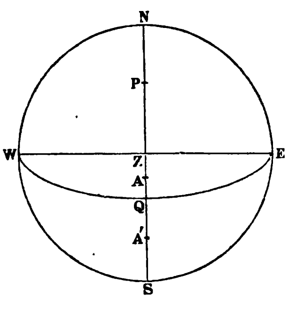
(98.) The meridian altitudes of a star were observed above and below the north pole to be 77° 8′ 10″ (zenith north of star), and 3° 40′ 45″ (zenith south), index correction + 1′ 42″, and height of eye 12 feet, required the latitude. Ans., lat. 53° 10′ 7″ N.

(99.) August 12, 1850, the meridian altitudes of a star were observed above and below the south pole to be 85° 14′ 15″ (zenith south), and 4° 52′ 0″ (zenith north), index correction — 8′ 14″, and height of eye above the sea was 30 feet, required the latitude.

Ans., lat. 49° 43′ 39″ S.

To find the latitude by the meridian altitude of a heavenly body above the pole, and the declination.

Let N W S E represent the horizon, N Z S the celestial meridian, P the north pole of the heavens, and W Q E the celestial meridian. Then Z Q is the latitude of spectator (p. 113, note).



First. Let the declination of the body and the latitude be of the same name, as when the body is at A; then A Q is the declination of the body, A S is its meridian altitude, and Z A its zenith distance, and Z Q = Z A + A Q;

or, lat. = meridian zenith distance + declination.

Second. Let the declination and latitude be of different names (one north and the other south); as when the body is at A′.

Then $ZQ = Z'A - 'AQ$;

or latitude = meridian zenith distance — declination.

When the zenith of the spectator is to the north of the body it is said to have a meridian altitude, *zenith north.* When the zenith is south of the body, its meridian altitude is called a meridian altitude, *zenith south.* By constructing figures similar to the above to suit different cases we shall find that the latitude is equal to the *sum* of the zenith distance and declination when the declination and zenith distance have the same names, namely, both north or both south and of the same name as either; and that the latitude is equal to the *difference* of the zenith distance and declination when they are of different names, and the latitude will be of the same name (N. or S.) as the greater.

Hence the following rules for finding the latitude from the meridian altitudes of different bodies.

Rule XXVIII.

To find the latitude by the meridian altitude of the sun, and its declination.

1. Find a Greenwich date in apparent time.

2. By means of the Nautical Almanac find the sun's declination for this date (p. 74). Take out also the sun's semidiameter, which is to be added to apparent altitude when lower limb is observed, and subtracted when upper limb is observed.

3. Correct the observed altitude for index correction, dip, semidiameter, and refraction and parallax, and thus get the true altitude (p. 109), subtract the true altitude from 90°, the result will be the true zenith distance.

4. Mark the zenith distance N. or S. according as the zenith is north or south of the sun.

5. Add together the declination and zenith distance if they have the *same* names; but take the difference if their

names be unlike; the result in each case will be the latitude, in the former of the name of either, in the latter of the name of the greater.

EXAMPLE.

April 27, 1853, in long. 87° 42′ W., the observed meridian altitude of the sun's lower limb was 48° 42′ 30″ (zenith north), the index correction was + 1′ 42″, and the height of eye above the sea was 18 feet, required the latitude.

Ship, April 27 . .	0^h	0^m
Long. in time . . .	5	51 W.
Greenwich, April 27	5	51

Sun's declination (at app. noon).

27	13°	43′	53″	N.
28	14	2	57	N.
		19	4	
·61306				
·97500				
1·58806 . .		4	38	
Sun's declin. . .	13	39	15	N.

Obs. alt. . . .	48°	42′	30″	
Index cor. . . .		1	42	+
	48	44	12	
Dip		4	11	—
	48	40	1	
Semi.		15	54	+
App. alt. centre .	48	55	55	
Cor. in alt. . .			45	—
True alt. . . .	48	55	10	
	90			
True zen. dist. .	41	4	50	N.
Declination . .	13	39	15	N.
Latitude . . .	54	44	5	N.

(100.) January 14, 1853, in long. 72° 42′ W. the observed meridian altitude of the sun's L. L. was 32° 42′ 10″ (Z. N.) the index correction + 2′ 10″, and height of eye above the sea 14 feet, required the latitude.

Ans., lat. 35° 50′ 34″ N.

(101.) March 20, 1853, in long. 72° 42′ E. the observed meridian altitude of the sun's L.L. was 45° 4′ 20″ (Z. S.),

index correction — 3′ 4″, and height of eye above the sea 20 feet, required the latitude. Ans., lat. 44° 56′ 54″ S.

(102.) July 4, 1853, in long. 100° 0′ W. the observed meridian altitude of the sun's L. L. was 62° 8′ 7″ (Z. N.), index correction — 3′ 0″, and height of eye above the sea 15 feet, required the latitude. Ans., lat. 50° 34′ 59″ N.

(103.) March 21, 1853, in long. 62° 0′ W., the observed meridian altitude of the sun's U. L. was 50° 10′ 5″ (Z. N.), index correction + 7′ 10″, and height of eye 14 feet, required the latitude. Ans., lat. 40° 26′ 47″ N.

(104.) Sept. 24, 1853, in long. 33° 0′ E., the observed meridian altitude of the sun's U. L. was 42° 3′ 15″ (Z. N.), index correction – 1′ 4″, and height of eye above the sea 18 feet, required the latitude.

Ans., lat. 47° 49′ 39″ N.

(105.) June 3, 1853, in long. 178° 30′ W., the observed meridian altitude of the sun's U. L. was 16° 20′ 0″ (Z. S.), index correction + 3′ 30″, and height of eye above the sea 20 feet, required the latitude.

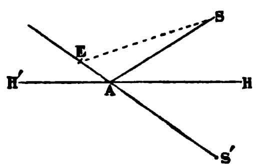
Ans., lat. 51° 35′ 39″ S.

Elements from Nautical Almanac.

	Sun's declination at apparent noon.		Sun's semidiameter.
Jan.	14 . . 21° 16′ 4″ S.	15 . . 21° 5′ 7″ S.	14 . . 16′ 18″
March	19 . . 0 27 54 S.	20 . . 0 4 13 S.	19 . . 16 5
July	4 . . 22 53 8 N.	5 . . 22 47 39 N.	4 . . 15 46
March	21 . . 0 19 28 N.	22 . . 0 43 7 N.	21 . . 16 5
Sept.	23 . . 0 8 3 S.	24 . . 0 31 28 S.	23 . . 15 59
June	3 . . 22 20 42 N.	4 . . 22 27 50 N.	3 . . 15 48

When the altitude of a heavenly body is observed by means of an artificial horizon, the reading off on the instrument will be the angular distance between the heavenly body and its image in the artificial horizon, and this will be double the altitude as observed from the true horizon. This will be easily seen by the following figure. Let S A, a ray

of light proceeding from the body at S, be reflected by means of an artificial horizon placed at A, in the line A E. Then, if the spectator's eye is in the line A E, as at E, the image of the body will appear in the direction E A coming from a point S′ below the horizon A. Now the observer is supposed to be placed so near A that the distance E A is inappreciable when compared with the distance A S of the heavenly body,



that is, the angle observed between S and S′, namely, S E S′ may be considered to be = S A S′ and this angle S A S′ is manifestly double S A H, the altitude above the horizontal plane H H. For by the principles of optics it is proved that the angle S A H is equal to E A H, which is equal to the vertical or opposite angle S′ A H, that is, the horizontal line A H bisects the angle observed. Hence the following rule for finding the true altitude from an observed altitude in the artificial horizon.

Rule XXIX.

Given, the observed altitude of a heavenly body in an artificial horizon, to find the true altitude.

1. Correct the observed altitude for index correction.

2. Half of the result will be the apparent altitude of the point observed.

3. Then proceed as in the preceding rules to find the true altitude.

EXAMPLES.

1. The observed altitude of the sun's lower limb in an artificial horizon was 98° 14′ 10,″ index correction — 4° 10′ required, apparent altitude of sun's lower limb.

Observed altitude . . .	98°	14′	10″
Index correction . . .		4	10 —
	2)98	10	0
Appar. alt. sun's l. limb.	49	5	0

2. Oct. 21, 1853, in long. 1° 6′ W., observed the meridian altitude of the sun's lower limb (in quicksilver horizon) to be 56° 14′ 0″ (Z. N.), index correction — 0′ 10″, required the latitude.

Ship, Oct. 21.	0^h	0^m
Long. in time . . .		4 W.
Greenwich, Oct. 21. . .	0	4

Sun's decl. (for app. noon.)

21	10°	35′	22″	S.
22	10	56	45	S.
		21	23	
.2·55630				
·92520				
3·48150 . .			3	
Sun's declin. . .	10	35	25	S.

Obs. alt. . . .	56°	14′	0″
In cor.		0	10 —
	2) 56	13	50
	28	6	55
Semi.		16	6 +
App. alt. centre	28	23	1
Cor. in alt. . .		1	40 —
True alt. . . .	28	21	21
	90		
True zen. dist. .	61	38	39 N.
Declination . .	10	35	25 S.
Latitude . . .	51	3	14 N.

(106.) Oct. 9, 1853, in long. 19° 20′ W., the observed meridian altitude of the sun's lower limb (in artificial horizon) was 44° 30′ 15″ (Z. S.), index correction — 2′ 10″, required the latitude. Ans., lat. 73° 53′ 28″ S.

(107.) June 10, 1853, in long., 23° 40′ E. the observed meridian altitude of the sun's lower limb (in quicksilver horizon) was 72° 15′ 20″ (Z. N.), index correction + 4′ 5″, required the latitude. Ans., lat. 76° 37′ 45″ N.

(108.) Aug. 7, 1853, in long. 62° 11′ E., the observed meridian altitude of sun's lower limb (in artificial horizon)

was 83° 30′ 0″ (Z. N.), the index correction — 3′ 15″, required the latitude. Ans., lat. 65° 0′ 22″ N.

(109.) May 3, 1853 in long. 14° 20′ W. the observed meridian altitude of sun's upper limb (in artificial horizon) was 30° 2′ 30″ (Z. S.), index correction — 1′ 15″, required the latitude. Ans., lat. 59° 34′ 14″ S.

(110.) July 17, 1853, in long. 72° 30′ E., the observed meridian altitude of sun's upper limb (in artificial horizon) was 52° 30′ 0″ (Z. N.), index correction + 2′ 10″, required the latitude. Ans., lat. 85° 15′ 16″ N.

Elements from Nautical Almanac.

	Sun's declination.		Sun's semi.
Oct.	9 . . 6° 20′ 10″ S.	10 . . 6° 42′ 58″ S.	9° . . 16′ 4″
June	9 . . 22 57 36 N.	10 . . 23 2 20 N.	9 . . 15 46
Aug.	6 . . 16 40 46 N.	7 . . 16 24 4 N.	6 . . 15 48
May	3 . . 15 43 50 N.	4 . . 16 1 18 N.	3 . . 15 53
July	16 . . 21 21 46 N.	17 . . 21 11 43 N.	16 . . 15 46

Rule XXX.

To find the latitude by the meridian altitude of the moon, and its declination, &c.

Since the moon's declination, &c., are given in the Nautical Almanac, for Greenwich *mean* noon, we must get a Greenwich date in *mean* time.

1. Find a Greenwich date in mean time.

2. By means of the Nautical Almanac find for this date the moon's declination, moon's semidiameter, and moon's horizontal parallax, augmenting the moon's semidiameter for altitude. (Rules X., XII.)

3. Correct the observed altitude for index correction, dip, semidiameter, and parallax and refraction, and thus get the true altitude; subtract the true altitude from 90°, and thus get the true zenith distance.

4. Mark the zenith distance N. or S. according as the zenith is north or south of the moon.

5. Add together the declination and zenith distance, if they have the same names, but take their difference if their names be unlike, the result in each case will be the latitude, in the former of the name of either, in the latter of the name of the greater.

EXAMPLES.

November 12, 1853, at $2^h\ 20^m$ P.M., mean time nearly, in long. 60° 42′ W. observed the meridian altitude of the moon's lower limb to be 30° 30′ 40″ (Z. N.), the index correction + 10′ 42″, and height of eye above the sea 16 feet, required the latitude.

Nov. 12, at	2^h	20^m	
Long.	4	3	+
Greenwich, Nov. 12 . .	6	23	

Moon's declination.		Moon's semi.		Hor. par.	
Nov. 12, at 6^h . .	2° 44′ 20″ N.	Noon . .	15′ 6″·4 . . .		55′ 19″·7
„ 7 . .	2 57 38 N.	Mid. . .	15 2·7 . . .		55 6·4
	13 18		3·7		13·3
·41642		·27413		·27413	
·65430		3·46522		2·90957	
logis. log. 1·07072	5 6	3·73935	2·0	3·18370	7·1
Decl.	2 49 26 N.		15 4·4		55 12·6
		Aug. .	7·4 +		
			15 11·8		

Moon's alt.	30°	30′	40″	
in cor.		10	42	+
	30	41	22	
Dip		3	56	—
	30	37	26	
Semi.		15	12	+
	30	52	38	
Cor. in alt. . . . . . {		45	36	
			10	
True alt..	31	38	24	
Zenith dist.	58	21	36	N.
Declin.	2	49	26	N.
Latitude	61	11	2	N.

(111.) Jan. 10, 1853, at $7^h 40^m$ P.M, mean time nearly, in long. 5° 30′ E., the observed meridian altitude of the moon's lower limb was 10° 20′ 30″ (Z. N.), the index correction — 2′ 20″, and height of eye 14 feet, required the latitude.

Ans., lat. 56° 37′ 46″ N.

(112.) Feb. 4, 1853, at $5^h 40^m$ A.M., mean time nearly, in long. 72° 18′ W., the observed meridian altitude of the moon's lower limb was 40° 20′ 15″ (Z. N.), index correction + 3′ 40″, and height of eye 15 feet, required the latitude.

Ans., lat. 25° 17′ 10″ N.

(113.) March 7, 1853, at $3^h 20^m$ P.M., mean time nearly, in long. 19° 20′ W., the observed meridian altitude of the moon's lower limb was 19° 17′ 18″ (Z. S.), index correction — 1′ 15″, and height of eye 16 feet, required the latitude.

Ans., lat. 88° 0′ 44″ S.

(114.) July 5, 1853, at $1^h 7^m$ P.M., mean time nearly, in long. 33° 30′ E., the observed meridian altitude of the moon's upper limb was 25° 42′ 30″ (Z. N.), the index correction + 2′ 15″, and height of eye 20 feet, required the latitude.

Ans., lat. 88° 22′ 37″ N.

(115.) Aug. 12, 1853, at $5^h 4^m$ A.M., mean time nearly, in long. 94° 40′ E., the observed meridian altitude of the moon's upper limb was 72° 20′ 0″ (Z. S.), the index correction + 3′ 40″, and height of eye 22 feet, required the latitude.

Ans., lat. 31° 53′ 3″ S.

(116.) Dec. 27, 1853, at $9^h 12^m$ A.M., mean time nearly, in long. 15° 20′ W., the observed meridian altitude of the moon's upper limb was 19° 50′ 4″ (Z. S.), the index correction — 0′ 30″, and height of eye above the sea was 24 feet, required the latitude. Ans., lat. 87° 35′ 20″ S.

Elements from Nautical Almanac.

	Moon's declination.	Moon's semi.	Hor. par.
Jan. 10, at 7^h	22° 1′ 16″ S.	noon 16′ 0″·1	58′ 36″·4
„ „ 8	21 55 8 S.	mid. 15 54 ·2	58 14 ·9
Feb. 3, at 22	23 20 51 S.	mid. 16 8 ·2	59 6 ·1
„ „ 23	23 24 43 S.	noon 16 6 ·6	59 0 ·3
Mar. 7, at 4	18 25 4 S.	noon 15 33 ·4	56 58 ·5
„ „ 5	18 15 51 S.	mid. 15 29 ·3	56 43 ·7
July 4, at 22	24 33 11 N.	mid. 14 50 ·7	54 22 ·0
„ „ 23	24 35 27 N.	noon 14 52 ·9	54 30 ·1
Aug. 11, at 10	14 4 13 S.	noon 16 6 ·9	59 1 ·1
„ „ 11	14 16 46 S.	mid. 16 9 ·2	59 9 ·7
Dec. 26, at 22	17 55 16 S.	mid. 16 27 ·4	60 16 ·7
„ „ 23	18 7 6 S.	noon 16 33 ·2	60 37 ·6

Rule XXXI.

To find the latitude by the meridian altitude of a fixed star, and its declination.

The declination of a fixed star changes so slowly, that we may, without any practical error, take it out of the Nautical Almanac by *inspection;* a Greenwich date will therefore be unnecessary.

1. Correct the observed altitude for index correction, dip, and refraction, and thus get the true meridian altitude: subtract this from 90° to obtain the true zenith distance.

2. Mark the same N. or S. according as the star is north or south of the zenith.

3. Take out the star's declination by inspection from the Nautical Almanac, and apply it to the true zenith distance in the manner pointed out in Rule XXVIII., Art. 5, and thus get the latitude.

EXAMPLES.

Feb. 10, 1853, the observed meridian altitude of *α* Hydræ was 35° 50′ 40″ (zenith north of star), the index correction

was + 2′ 10″, and height of eye 0 feet, required the latitude.

Observed altitude	35°	50′	40″	
Index correction .		2	10	+
	35	52	50	
Refraction . . .		1	14	—
True altitude . .	35	51	36	
	90			
True zenith dist. .	54	8	24	N.
Declination . .	8	1	29	S. (Naut. Alm. p. 455.)
Latitude . . .	46	6	55	N.

(117.) May 21, 1853, the observed meridian altitude of α Bootis was 62° 42′ 10″ (Z. N.), the index correction — 4′ 4″, and height of eye 18 feet, required the latitude.

Ans., lat. 47° 23′ 32″ N.

(118.) June 16, 1853, the observed meridian altitude of α Lyræ was 77° 1′ 50″ (Z. N.), index correction + 2′ 10″, and height of eye 16 feet, required the latitude.

Ans., lat. 51° 39′ 4″ N.

(119.) May 6, 1853, the observed meridian altitude of α Virginis was 16° 52′ 5″ (Z. N.), index correction + 1′ 45″, and height of eye 20 feet, required the latitude.

Ans., lat. 62° 50′ 4″ N.

(120.) Oct. 26, 1853, the observed meridian altitude of α Piscis Australis was 70° 10′ 0″ (Z. S.), the index correction — 4′ 5″, and height of eye 10 feet, required the latitude.

Ans., lat. 50° 21′ 23″ S.

(121.) May 10, 1853, the observed meridian altitude of α^2 Centauri was 10° 4′ 15″ (Z. N.), index correction — 2′ 10″, and height of eye 20 feet, required the latitude.

Ans., lat. 9° 54′ 9″ N.

(122.) Aug. 1, 1853, the observed altitude of α Aquilæ was 50° 4′ 15″ (Z. N.), index correction — 4′ 10″, and height of eye 14 feet, required the latitude.

Ans., lat. 48° 33′ 32″ N.

Elements from Nautical Almanac.

May 21 .	α Bootis . . .	Decl. 19°	56′	57″	N.
June 16 .	α Lyræ . . .	„ 38	38	55	N.
May 6 .	α Virginis . .	„ 10	23	40	S.
Oct. 26 .	α Piscis Australis	„ 30	23	53	S.
May 10 .	α^2 Centauri . .	„ 60	13	31	S.
Aug. 1 .	α Aquilæ . . .	„ 8	29	7	N.

Rule XXXII.

To find the latitude by the meridian altitude of a planet, and its declination.

1. Find a Greenwich date in mean time.

2. By means of the Nautical Almanac find the planet's declination for this date; and when great accuracy is required take out the planet's semidiameter and horizontal parallax.

3. Correct the observed altitude for index correction, dip, refraction (and if necessary for semidiameter and parallax in altitude), and thus get the true altitude. Subtract the true altitude from 90° to get the true zenith distance.

4. Mark the zenith distance north or south according as the zenith is north or south of the planet.

5. Proceed then as in Rule XXVIII., Art. 5.

EXAMPLE.

November 20, 1853, at $6^h\ 18^m$ A.M., mean time nearly, in long. 62° 42′ E. observed the meridian altitude of Mars' lower limb to be 52° 10′ 45″ (Z. N.), the index correction + 4′ 0″, and height of eye above the sea 16 feet, required the latitude.

Ship, Nov. 19 . . .	$18^h\ 18^m$	Planet's semi.	3″
Long. in time . . .	4 11 E	„ H.P.	6
Greenwich, Nov. 19 .	14 7		

Planet's declination.				
19	12°	55′	36″	N.
20	12	47	1	N.
		18	35	
·23048				
·98615				
1·21663 . .		10	56	
Planet's decl. .	12	44	40	N.

Obs. alt. . . .	52°	10′	45″	
In. cor. . . .		4	0	+
	52	14	45	
Dip.		3	56	
	52	10	49	
Semi.			3	
	52	10	52	
Refr.			45	—
	52	10	7	
Par. in alt. . .			4	+
True alt. . . .	52	10	11	
	90			
True zen. dist. .	37	49	49	N.
Planet's decl. .	12	44	40	N.
Latitude . . .	50	34	29	N.

If the small corrections of the planet's semidiameter and parallax in altitude are neglected, the above example will be worked thus:

Ship, Nov. 19	18^h	18^m
Long. in time	4	11 E
Greenwich, Nov. 19 . .	14	7

Planet's declination.				
19	12°	55′	36″	N.
20	12	47	1	N.
		18	35	
·23048				
·98615				
1·21663		10	56	
Planet's declin. .	12	44	40	N.

Obs. alt. . . .	52°	10′	45″	
In. cor. . . .		4	0	+
	52	14	45	
Dip		3	56	
	52	10	49	
Refr.			45	—
True alt. . . .	52	10	4	
	90			
True zen. dist. .	37	49	56	N.
Declin. . . .	12	44	40	N.
Latitude . . .	50	34	36	N.

EXAMPLES.

(123.) May 4, 1853, at 2^h 45^m A.M., mean time nearly, in long. 42° 10′ W., the observed meridian altitude of Jupiter's

centre was 16° 42′ 10″ (Z. N.), index correction + 11′ 42″, and height of eye above the sea 20 feet, required the latitude.

Ans., lat. 50° 30′ 38″ N.

(124.) July 12, 1853, at 9^h 36^m P.M., mean time nearly, in long. 30° 30′ E., the observed meridian altitude of Jupiter's centre was 10° 10′ 50″ (Z. N.), the index correction — 4′ 4″, and height of eye above the sea 10 feet, required the latitude. Ans., lat. 57° 45′ 37″ N.

(125.) November 27, 1853, at $6^h \cdot 3^m$ A.M., mean time nearly, in long. 100° 0′ W., the observed meridian altitude of Mars' centre was 32° 40′ 10″ (Z. S.), index correction — 8′ 10″, and height of eye 16 feet, required the latitude.

Ans., lat. 45° 45′ 0″ S.

(126.) Sept. 15, 1853, at 4^h 20^m A.M., mean time nearly, in long. 10° 6′ W., the observed meridian altitude of Saturn's centre was 19° 42′ 10″ (Z. N.), index correction — 6′ 45″, and height of eye 12 feet, required the latitude.

Ans., lat. 88° 55′ 24″ N.

(127.) Jan. 12, 1853, at 7^h 9^m P.M., mean time nearly, in long. 32° 0′ W., the observed meridian altitude of Saturn's centre was 62° 42′ 10″ (Z. S.), index correction — 8′ 10″, and height of eye 20 feet, required the latitude.

Ans., lat. 14° 36′ 41″ S.

(128.) June 7, 1853, at 5^h 40^m P.M., mean time nearly, in long 72° 30′ E., the observed meridian altitude of Venus was 30° 40′ 10″ (Z. S.), index correction + 4′ 20″, and height of eye 24 feet, required the latitude.

Ans., lat. 35° 39′ 30″ S.

Elements from Nautical Almanac.

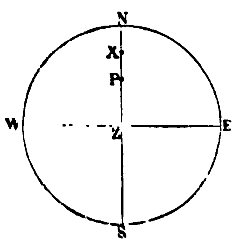
	Planet's declination.		
May 3 . .	22° 43′ 11″	S.	Jupiter.
„ 4 . .	22 43 1		
July 12 . .	22 16 10	S.	Jupiter.
„ 13 . .	22 15 49		

	Planet's declination.				
Nov. 27 . .	11	48	44	N.	Mars.
„ 28 . .	11	40	44		
Sept. 14 . .	18	24	50	N.	Saturn.
„ 15 . .	18	24	38		
Jan. 12 . .	12	54	5	N.	Saturn.
„ 13 . .	12	54	24		
June 7 . .	23	42	15	N.	Venus.
„ 8 . .	23	48	1		

Rule XXXIII.

To find the latitude by the meridian altitude of a heavenly body below the pole, and the declination.

Let X be the place of a heavenly body on the meridian, below the pole of the heavens P; then NX = meridian altitude below pole, PX = polar distance = 90° — declination. Now, PN = altitude of pole = latitude of spectator, whose zenith is Z, and PN or latitude = NX + PX = altitude + 90° — declination. Hence this rule for finding the latitude from the meridian altitude below the pole.



1. Find the declination of the heavenly body at the time of observation.

2. From the observed altitude get the true altitude.

3. Add 90° to the true altitude, and from the sum subtract the declination; the remainder will be the latitude.

EXAMPLES.

1. April 27, 1853, the meridian altitude of *a* Crucis below the south pole was observed to be 14° 10′ 30″, the index correction was + 4′ 4″, and the height of eye 20 feet, required the latitude.

Observed altitude . .	14°	10′	30″
Index correction . .		4	4 +
	14	14	34
Dip		4	24 —
	14	10	10
Refraction		3	47 —
True altitude . . .	14	6	23
	90		
	104	6	23
Star's declination . .	62	17	10
Latitude	41	49	13 S.

2. June 18, 1853, at apparent midnight, in long. 100° W., the observed meridian altitude of the sun's lower limb below the north pole was 8° 42′ 10″, the index correction — 3′, and height of eye above the sea 14 feet, required the latitude.

Ship, June 18 . .	12^h	0^m
Long. in time . .	6	40 W.
Greenwich, June 18	18	40

Sun's declination (app. noon).

18	23°	25′	36″ N.
19	23	26	39 N.
		1	3
·10915			
2·23408			
2·34323 . .		0	49
Sun's declin. . .	23	26	25 N.

Obs. alt. . . .	8°	42′	10″
In. cor. . . .		3	0 —
	8	39	10
Dip.		3	41 —
	8	35	29
Semi.		15	46 +
	8	51	15
Cor. in alt. . .		5	51 —
	8	45	24
	90		
	98	45	24
Sun's declin. .	23	26	25 N.
Latitude . . .	75	18	59 N.

(129.) Feb. 10, 1853, the meridian altitude of α Argûs below the pole was observed to be 6° 41′ 15″, index correction — 2′ 10″, and height of eye above the sea 14 feet, required the latitude. Ans., lat. 43° 50′ 18″ S.

(130.) January 11, 1853, the observed meridian altitude of α Ursæ Majoris, below the pole, was 14° 14′ 30″, the index correction — 4′ 5″, and height of eye 20 feet, required the latitude. Ans., lat. 41° 29′ 47″ N.

(131.) April 20, 1853, the observed meridian altitude of η Argûs, below the pole was 20° 14′ 15″, the index correction — 4′ 5″, and the height of eye 10 feet, required the latitude. Ans., lat. 51° 9′ 27″ S.

(132.) June 1, 1853, in long. 30° 52′ W., the observed meridian altitude of the sun's lower limb, below the pole, was 10° 42′ 0″, the index correction + 2′ 10″, and height of eye 20 feet, required the latitude. Ans., lat. 77° 41′ 0″ N.

(133.) June 10, 1853, at $2^h\ 40^m$ A.M., mean time nearly, in long. 30° W., observed the meridian altitude of the moon's lower limb, below the pole, to be 14° 30′ 10″, index correction + 2′ 45″, height of eye 14 feet, required the latitude. Ans., lat. 81° 32′ 31″ N.

(134.) July 1, 1853, at $9^h\ 30^m$ P.M., mean time nearly, in long. 62° W., the observed meridian altitude of Mars below the pole was 10° 32′ 30″, index correction — 3′ 0″, and height of eye 18 feet, required the latitude.

Ans., lat. 79° 8′ 32″ N.

Elements from Nautical Almanac.

		Star's declination.			
Feb. 10 .	α Argûs . . .	52°	36′	74″	S.
Jan. 11 .	α Ursæ Majoris	62	32	26	N.
April 20 .	η Argûs . . .	58	54	59	S.

	Sun's declination.		Sun's semidiameter.
June 1 . . .	22° 5′ 15″	N. . .	15′ 48″
„ 2 . . .	22 13 10		

	Moon's declination.	Moon's semi.	Hor. par.
June 9, at 16^h	24° 3′ 51″ N.	mid. 15′ 0″·4	54′ 57″·6
„ „ 17	24 0 11 N.	noon 15 4 ·0	55 11 ·0

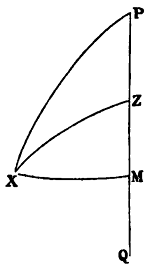
	Planet's declination.
July 1	21° 7′ 5″ N.
„ 2	21 15 9 N.

Latitude by observations off the meridian.

In the volume of astronomical problems* will be found several methods for finding the latitude depending on some particular bearing or hour angle of the heavenly body: as, when it bears due east, or, when it is in the horizon, or when the hour angle is 6 hours, &c.; but since it is difficult to determine the precise moment when the heavenly body is in either of these positions, the methods referred to are of little use in practice. Problem 131, however, is one from which a useful rule may be derived, as it depends on the declination, altitude and hour angle of the heavenly body. The altitude and declination are easily obtained at sea; the hour angle is only known accurately when the ship time is given, and this is a quantity somewhat difficult to discover independently of an observation: the ship time, however, may always be considered to be known nearly. To render therefore a rule for finding the latitude, depending on the declination, altitude and ship time of practical value, we must ascertain in what position of a heavenly body an error of a few minutes in the ship time will produce the smallest error in the latitude deduced from it: and this we find will be the case, if the observed altitude is taken when the body is *near the meridian*. It is for this reason that single altitude observations taken off the meridian for finding the latitude are confined to bodies within half an hour of the meridian, when the time at the ship is uncertain to 3 or 4 minutes.

* "Problems in Astronomy, &c., and Solutions," pp. 33, 34, &c.

To find the latitude from the hour angle, altitude, and declination of a heavenly body.



Let P Z Q be the celestial meridian, P the pole, Z the zenith, and X the place of a heavenly body.

Then in the triangle P X Z are given the hour angle P, the polar distance P X = 90 ± declination, and the zenith distance Z X = 90 — altitude, to find P Z the colatitude, and thence the latitude.

Investigation.

Let fall a perpendicular X M upon the meridian P Q, thus forming two right-angled spherical triangles P X M, Z X M.

Let P X $= p$, Z X $= a$, P $= h$.

also PM $= x$, ZM $= y$, XM $= z$.

Then colat. P Z $= x - y$, when the perpendicular X M does not fall between the pole and the zenith, and P Z $= x + y$, when the perpendicular does so fall, a position easy to discover by observation.

First. To find X or P M.

In triangle P X M . . $\cos. h = \cot. p \tan. x$

$\therefore \tan. x = \cos. h \tan. p.$

$=$ cos. hour angle cot. decl. (1).

Second. To find y or Z M.

In triangle X P M . . $\cos. p = \cos. z \,.\, \cos. x$

In triangle Z X M . . $\cos. a = \cos. y \,.\, \cos. z$

dividing, so as to eliminate cos. z . . $\dfrac{\cos. p}{\cos. a} = \dfrac{\cos. x}{\cos. y}$

$\therefore \cos. y = \cos. a \,.\, \cos. x \,.\, \sec. p.$

$= \cos. x$. sin. alt. cosec. decl. . . . (2).

Formulæ (1) and (2) determine x and y and thence the latitude, since colat. $= x \pm y$.

EXAMPLE.

Given, hour angle = $3^h\,5^m\,36^s$, declination = $10°\,54'\,26''$ N., and altitude = $35°\,4'\,7''$, to find $x - y$ the colatitude.

Tan. x = cot. d, cos. h.

Cot. d	0·715072
Cos. h	9·838610
Tan. x	10·553682
x =	74° 23′ 7″
y =	35 11 30
	39 11 37
	90
Latitude . . .	50 48 23 N.

Cos. y = cosec. d, sin. alt. cos. x.

Cosec. d	0·722991
Sin. alt.	9·759332
Cos. x	9·430025
Cos. y	9·912348
y =	35° 11′ 30″

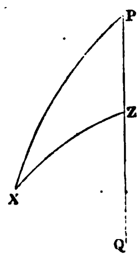
A rule deduced from the above formulæ is open to the objection of a distinction of cases. For, first, if the perpendicular X M fall upon P Z, between the pole and the zenith, then the colatitude P Z is equal to $x + y$ or the two arcs must be added, instead of being subtracted as in the above example. Secondly, If the declination and latitude are of different names then tangent x is negative: this is evident by a figure. To find x in the latter case the arc taken out of the tables must be subtracted from 180°. If the student is able to discover these distinctions, the above formula is valuable, as he can derive from it a useful and practical rule.

We will now proceed to give another rule, which is free from the objection mentioned above; this rule, however, will require the latitude to be known within a quarter of a degree of the truth, otherwise it may be necessary to repeat a part of the work, perhaps more than once.

To find the latitude from the altitude of a heavenly body near the meridian, *and its declination and hour angle.*

Let P be the pole, Z the zenith, P Z Q the celestial meridian, P Q = 90°, then Z Q = latitude of spectator.

Let X be the place of a heavenly body near the meridian.



Draw the circle of declination P X and circle of altitude Z X through X, then in the spherical triangle P Z X are given the hour angle P, the polar distance P X, and the zenith distance Z X, to find the colatitude P Z. This may be done by dropping a perpendicular from X upon P Q, in the manner pointed out in Problem 131 of the Astronomical Problems: but the direct method of solving it being long and tedious an analytical formula is obtained for this purpose (see astronomical problems, p. 201), from which the following rule is deduced.

Rule XXXIV.

To find the latitude from an altitude of the sun near the meridian.

1. Find the Greenwich date in mean time.
2. Take out the declination and equation of time for this date.
3. *To find the sun's hour angle.* To the Greenwich mean time found as accurately as possible apply the longitude in time; subtracting if west, and adding if east; the result will be ship mean time: to this apply the equation of time with its proper sign to reduce mean time into apparent time; the result will be the sun's hour angle.
4. Add together the following logarithms,—

Constant log., 6·301030
Log. cosine declination.
Log. cosine estimated latitude.
Log. haversine hour angle.*

reject 30 in the index, and look for the result as a logarithm, and take out its natural number.

* Or, instead of log. haversine, twice the log. sine of half the hour angle (rejecting in this case 40 from the index).

5. Correct the observed altitude for index correction, dip, semidiameter, correction in altitude, and thus get a zenith distance.

6. From the versine of zenith distance subtract the natural number found as above. The remainder will be the versine of a meridian zenith distance, which find from the tables.

7. Under the meridian zenith distance put the declination, and proceed to find the latitude by one of the preceding rules for finding the latitude by a meridian altitude.

NOTE.—If the latitude thus found differ much from the estimated latitude used in the question, the work should be corrected by using the last latitude found, in place of the former one.

EXAMPLES.

August 22, 1853, A.M., in latitude by account 50° 48′ N., and long. 1° 6′ W., a chronometer showed $11^h\ 50^m\ 22^s$, error on Greenwich mean time being $40{\cdot}2^s$ fast, when the observed altitude of the sun's lower limb (in artificial horizon) was 101° 14′ 10″ (Z. N.), index correction + 30″, required the latitude.

Chro. showed .	11^h	50^m	22^s	A.M.
Error of chro. .			40 ·2	fast
	11	49	41 ·8	
	12			
Gr. Aug. 21 .	23	50		

To find the hour angle.

Chro. showed . .	11^h	50^m	22^s	
Error on G. M. T.			40 ·2—	
G. M. T. 12^h +	11	49	41 ·8	
Long. in time .		4	24	W.
Ship M. T. . . .	23	45	17 ·8	
Eq. of time . .		2	39 ·0	
Hour angle . .	23	47	56 ·8	

	Sun's declination.				Equation of time.		
21	12°	4′	57″ N.	. . . .	2^m	54^s	add
22	11	44	50 N.	. . . .	2	39	
		20	7			15	
·00303				·00303			
·95172				2·85733			
·95475 . .		19	59	2·86036		15	
Declin. . . .	11	44	58 N.		2	39	

Obs. alt. in horiz.	101°	14′	10″
In. cor. . . .			30 +
2)	101	14	40
	50	37	20
Semi.		15	51
	50	53	11
Cor. in alt. . .			42 —
	50	52	29
	90		
Zen. dist. . . .	39	7	31
. . . arc. . .	39	2	51 N.
Declin. .	11	44	58 N.
Latitude.	50	47	49 N.

Constant log. . .	6·301030
Log. cos. dec. . .	9·990803
„ cos. est. lat. .	9·800737
„ hav. H. A. .	6·839449
	2·932019
Nat. No.	855
Vers. zen. dist. . .	224232
Nat. No.	855
Ver. mer. zen. dist.	223377
	220
	157

As this latitude differs from the estimated latitude, one part of the above operation should be repeated, using lat. 50° 47′ 49″ instead of 50° 48′, thus—

Constant log. . .	6·301030
Log. cos decl. . . .	9·990803
Log. hav. H. A. . .	6·839449
Log. cos. 50° 47′ 49″ .	9·800776
	2·932058
	855

The same natural number as before, which shows that the erroneous latitude used in the first operation produced no practical error in the resulting latitude.

The above example worked by formulæ, p. 134.

tan. x = cot. decl. cos. hour angle

cos. y = cosec. decl. sin. alt. cos. x.

Cot. decl. . .	0·681957
Cos. h	9·999399
Tan. x. . . .	10·681356
x . .	78° 14′ 5″

Cosec. decl. .	0·691153		
Sin. alt. . . .	9·889732		
Cos. x . . .	9·309423		
Cos. y . . .	9·890308		
y . .	39°	1′	54″
x . .	78	14	5
$x - y$.	39	12	11
	90		
Latitude .	50	47	49 N.

(135.) May 10, 1853, A.M., in latitude by account 50° 50′ N., and long. 2° 10′ W., a chronometer showed $11^h\ 51^m\ 58^s$, error on Greenwich mean time being $11^m\ 31^s$ fast, when the observed altitude of the sun's lower limb was 56° 19′ 30″ (Z. N.), index correction − 3′ 20″, and height of eye 18 feet, required the latitude.

Ans., lat. 50° 51′ 34″ N.

(136.) Nov. 14, 1853, P.M., in lat. by account 87° 41′ S. and long. 1° 0′ W., a chronometer showed $0^h\ 25^m\ 27^s$, error on Greenwich mean time being fast $5^m\ 56{\cdot}7^s$, when the observed altitude of the sun's lower limb was 20° 26′ 20″ (Z. S.), index correction − 2′ 20″, and height of eye 10 feet, required the latitude. Ans., lat. 87° 42′ 15″ S.

(137.) June 30, 1853, A.M., in lat. by account 63° 20′ N. and long. 23° 30′ W., a chronometer showed $11^h\ 30^m\ 15^s$, error on Greenwich mean time being $7^m\ 32^s$ fast, when the observed altitude of the sun's upper limb was 44° 20′ 22″ (Z. N.), index correction + 2′ 20″, and height of eye 14 feet, required the latitude. Ans., lat. 63° 21′ N.

(138.) July 10, 1853, A.M. in lat. by account 57° 24′ N. and long. 3° 40′ W., a chronometer showed $11^h\ 20^m\ 15^s$, error on Greenwich mean time being $30^m\ 30^s$ slow, when the observed altitude of the sun's lower limb was 54° 17′ 19″ (Z. N.), index correction — 2′ 40″, and height of eye 20 feet, required the latitude. Ans., lat. 57° 25′ 25″ N.

(139.) May 20, 1853, A.M., in lat. by account 79° 48′ N., and long. 44° 30′ E., a chronometer showed $11^h\ 30^m\ 0^s$, error on Greenwich mean time being $15^m\ 20^s$ slow, when the observed altitude of the sun's lower limb (in artificial horizon) was 54° 30′ 20″ (Z. N.), index correction — 4′ 30″, required the latitude. Ans., lat. 79° 48′ 30″ N.

(140.) June 16, 1853, P.M., in lat. by account 52° 25′ N., and long. 1° 6′ W., a chronometer showed $1^h\ 2^m\ 9^s$ error on Greenwich mean time being $40^m\ 30^s$ fast, when the observed altitude of the sun's lower limb was 60° 37′ 50″ (Z. N.),

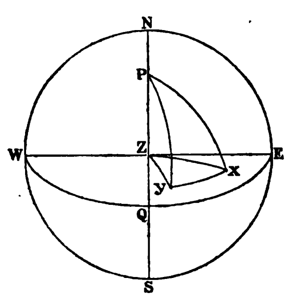
index correction — 2′ 10″, and height of eye 17 feet, required the latitude. Ans., lat. 52° 24′ 15″ N.

Elements from Nautical Almanac.

Sun's declination.					Equation of time.		Sun's semi.
May 9,	17°	24′	37″	N.	3^m 45^s·98	to be added	15′ 52″
„ 10,	17	40	25	N.	3 48·67		
Nov. 14,	18	18	5	S.	15 22·9	„ added	16 13
„ 15,	18	33	30	S.	15 12·8		
June 29,	23	14	31	N.	3 3·7	„ subtracted	15 46
„ 30,	23	11	3	N.	3 15·6		
July 9,	22	21	48	N.	4 50·8	„ subtracted	15 46
„ 10,	22	14	22	N.	4 59·5		
May 19,	19	48	45	N.	3 47·7	„ added	15 50
„ 20,	20	1	23	N.	3 44·9		
June 16,	23	22	15	N.	0 18·8	„ subtracted	15 46
„ 17,	23	24	8	N.	0 31·6		

To find the latitude by Inman's rule for double altitude.

The most general rule for finding the latitude by a double altitude of a heavenly body is the one selected by Dr. Inman: the labour of reducing the observations is somewhat greater than in the one known as Ivory's Rule, which follows: but the great advantage of the method adopted by Inman is that it may be applied to the same or different heavenly bodies, observed at the same instant or at different times: we will give examples of its application to all the cases that usually occur, referring the student for more complete information on the subject to the Appendix to "Inman's Navigation."



Let P be the pole, Z the zenith, X and y the same heavenly body observed at different times; or different heavenly bodies observed at the same instant, or different heavenly bodies

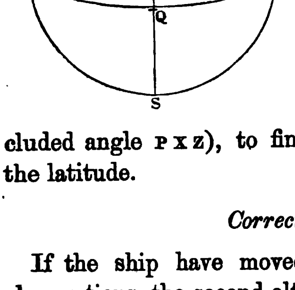
observed at different times. Let ZX Zy be their zenith distances. Then in the figure we know by observation Z X and Z y, and from the Nautical Almanac we can find the polar distances P X and Py; also by means of the elapsed time as measured by a watch, or from the right ascension of the bodies, or from both, we can compute the polar angle X P y; the colatitude P Z may then be computed in the following manner by the application of the common rules of spherical trigonometry.

1. In triangle PyX are given two sides P X, Py and the included angle X P y to find X y, which call arc 1.

2. In triangle P X y are given three sides P X, Py and arc 1, to find angle P X y, which call arc 2.

3. In triangle Z X y are given three sides Z X, Z y and arc 1, to find angle Z X y, which call arc 3.

4. Arc 2 — arc 3 = angle P X Z = arc 4. But if the arc Xy drawn through X and y pass when produced between P and Z the pole and the zenith, then it is evident by the annexed figure that the arc 2 + arc 3 = P X Z or arc 4. If the arc X y produced pass near Z, the bodies X and y in such a position should not be observed.



Lastly. In triangle P X Z are given the two sides P X and Z X and arc 4 (namely, the included angle P X Z), to find P Z the colatitude, and thence the latitude.

Correction for run.

If the ship have moved in the interval between the observations, the second altitude will in general differ from what it would have been if both observations had been taken at the same place. On this account it is usual to apply to the first altitude a correction so as to reduce it to

what it would have been if taken at the place of the second observation; this quantity is called "the correction for run of the ship," and may be calculated as follows.

When a ship describes an arc on the surface of the sea, the zenith describes a similar arc in the celestial concave: let, therefore, Z be the zenith of the ship at the first observation, Z′ its zenith at the second observation; then arc Z Z′ measures the distance run in the interval. Let S be the place of the heavenly body at the first observation: with centre S at distance S Z′, describe an arc cutting S Z, fig. 1, or S Z produced in D, fig. 2; then the triangle Z Z′ D being small, may be considered as a right-angled plane triangle, and Z D is the correction to be applied to Z S in order to get Z′ S the distance of S from the zenith at the second observation.

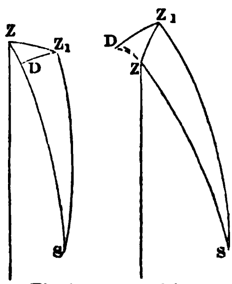


Fig. 1. Fig. 2.

Now Z D = Z Z′ cos. Z′ Z D

= distance run × cos. angle between the direction of the ship's run and the bearing of the sun at the first observation.

This correction Z D may be readily found by means of the traverse table, for since (Astronomical Problems, p. 122),

Diff. lat. = dist. cos. course; if therefore in triangle Z Z′ D the angle Z Z′ D be considered as the course, and Z Z′ the distance, the correction Z D for run will correspond in the table to the difference of latitude.

The angle Z′ Z D is the difference between the course of the ship in the interval and the true bearing of the body, when the run of the ship has been towards the place of the body, as in fig. 1; and what this angle wants of 180° or 16 points when the direction of the ship's run has been from the place of the body, as in fig. 2. In the former case it is manifest that the correction Z D for run must be

added to the first observed altitude, and in the second subtracted, in order to get the altitude of the body, the same as it would have been if it had been also observed at the place of the ship at the second observation.

Rule XXXV. (*for run*).

1. Enter the traverse table with the distance run as a distance, and the angle (supposed less than 8 points) between the true bearing of the heavenly body at the first observation and course of the ship, as a course, and take out the corresponding *diff. lat.*, which *add* to the first true altitude (the tenths in the diff. lat. being turned into seconds, by multiplying them by 60); the result will be the altitude corrected for run.

2. But if the above angle be greater than 8 points, subtract the same from 16 points, and look out the remainder as a course, and *subtract* the diff. lat. corresponding thereto from the first true altitude; the result will be the altitude corrected for run.

EXAMPLES.

1. The course of the ship was N.W. ½ W. 10 miles, and bearing of the sun E. by S., required the correction for the first altitude for run.

The angle between N.W. ½ W. and E. by S. is 13½ points, snbtracting 13½ from 16 points: enter traverse table with the remainder, namely, 2½ as a course and 10 miles as a distance: the corresponding diff. lat. is $8'{\cdot}8 = 8'\,48''$ to be *subtracted* from the true altitude.

2. The course of the ship was E.N.E. 25 miles, and bearing of the sun E. by S., required the correction of the first altitude for run.

The angle between E.N.E. and E. by S. is 3 points; entering traverse table with 3 points as a course, and 25 miles as a distance, the corresponding diff. lat. $= 28'{\cdot}8 = 20'\,48''$ to be *added* to the true altitude.

(141.) The true course of the ship was S.W.½W. 15 miles, and the true bearing of the sun S. by E. ½ E., required the correction of the first altitude for run. Ans., + 5′ 42″.

(142.) The true course of the ship was W. ½ N. 19 miles, and the true bearing of the sun was S. by E. ½ E., required the correction for run. Ans., — 7′ 18″.

Rules for finding the latitude by double altitude.

Rule XXXVI.

First. When the object observed is the sun.

1. From the estimated mean time at the ship at each observation, and the longitude, get two Greenwich dates.

2. By means of the Nautical Almanac find the declination for each Greenwich date. Take out also from the Almanac the sun's semidiameter.

3. Find the polar distance at each observation by subtracting the declination from 90°, if the estimated latitude and declination are of the same name; or by adding 90° to the declination, if the estimated latitude and declination are of different names.

4. Correct the two observed altitudes for index correction, dip, semidiameter, and correction in altitude.

5. Correct also the first altitude observed for the run of the ship (p. 143).

6. Subtract the true altitudes thus obtained from 90° and thus get the zenith distances.

7. Find the polar angle or elapsed time between the observations, by subtracting the time shown by chronometer at the first observation from the time shown by chronometer (increased if necessary by 12 hours) at second observation.

NOTE.—When great accuracy is required, this elapsed time should be corrected for rate of chronometer, and also for the change in the equation of time in the interval; but these corrections are seldom made.

8. *To find arc* 1 (using Inman's Tables). Add together

log. sin. polar distance at greater bearing, log. sin. polar distance at lesser bearing, and log. haversine of polar angle; reject 10 in the index and look out the result as a log. haversine; the arc corresponding thereto is arc 1 nearly.

9. *To find arc* 2. Under arc 1 put polar distance at greater bearing, and take the difference, under which put polar distance at lesser bearing; take the sum and difference of the two last quantities. Add together the log. cosecants of the two first arcs put down, and halves of the log. haversines of the two last arcs put down; the sum, rejecting 10 in index, is the log. haversine of arc 2, which take from the Tables.

10. *To find arc* 3. Under arc 1 put zenith distance at greater bearing, and take the difference, under which put zenith distance at lesser bearing: take the sum and difference of the last two quantities.

Add together the log. cosecants of the two first arcs put down, and halve the log. haversines of the two last arcs put down; the sum, rejecting 10 in index, is the log. haversine of arc 3, which take from the Tables.

11. *To find arc* 4. The difference between arc 2 and arc 3 is arc 4.

NOTE.—When the arc joining the places of the sun at the two observations passes, when produced, between the zenith and pole (which the observer may easily discover at the time the observation is taken), then the sum of arcs 2 and 3 is arc 4.

12. *To find arc* 5. Add together log. sin. polar dist. at greater bearing, log. sin. zenith distance at greater bearing and log. haversine of arc 4, the sum, rejecting 10 in the index, is log. haversine of arc, which take from the Tables, and call arc 5.

Take the difference between the polar distances at the greater bearing, and the zenith distance at greater bearing.

Add together versine of arc 5 and versine of the difference just found; the sum is the versine of the colatitude,

which take from the Tables, and subtract from 90°; the result is the latitude required.

EXAMPLE.

Oct. 11, 1845, in latitude by account 54° N. and long. 83° 15′ W., the following double altitude of the sun was observed.

Mean time nearly.	Chro.	Alt. sun's L. L.	Bearing.
7h 45m A.M.	11h 40m 15s	9° 0′ 20″	E.S.E. ¼ E.
10 35 A.M.	2 13 20	25 3 30	S.S.E.

The run of the ship in the interval was S. by W. 15 miles, index correction + 5′ 10″ and the height of eye above the sea was 18 feet: required the true latitude at the second observation.

At greater bearing.		At less bearing.	
Ship, Oct. 10 . .	19h 45m	Ship, Oct. 10 . .	22h 35m
Long. in time . .	5 33 W.		5 33 W.
Oct. 10	25 18	Oct. 10	28 8
Gr. Oct. 11 . . .	1 18	Gr. Oct. 11 . . .	4 8

Decl. at greater bearing.		Decl. at less bearing.	
11	7° 4′ 38″ S.	11	7° 4′ 38″ S.
12	7 27 15 S.	12	7 27 15 S.
	22 37		22 37
1·26627		·76391	
·90084		·90084	
2·16711 cor.	1 14	1·66475 cor.	3 54
	7 5 52 S.		7 8 32 S.
	90		90
N. Pol. dist. . .	97 5 52	N. Pol. dist. .	97 8 32
	At greater bearing.		At less bearing.

Sun's altitude at greater bearing.

Obs. alt. . . .	9°	0′	20″	
In. cor. . . .		5	10	+
	9	5	30	
Dip		4	11	—
	9	1	19	
Semi.		16	3	+
	9	17	22	
Refr.		5	33	—
	9	11	49	
Cor. for run. .		2	12	+
True alt. . . .	9	14	1	
	90			
Z. D. . . .	80	45	59	

At greater bearing.

Sun's altitude at less bearing.

Obs. alt. . . .	25°	3′	30″	
In. cor. . . .		5	10	+
	25	8	40	
Dip.		4	11	—
	25	4	29	
Semi.		16	3	+
	25	20	32	
Refr.		1	54	—
True alt. . . .	25	18	38	
	90			
Z. D.	64	41	22	

At less bearing.

To find arc 1.

Chro. times.

	11ʰ	40ᵐ	15ˢ
	14	13	20
Pol. angle . .	2	33	5

Sin. P. D. at G. B. .	9·996661
Sin. P. D. at L. B. .	9·996617
Log. hav. pol. angle	9·031223
Hav. arc 1 . . .	9·024501
Arc. 1 . .	37° 58′ 0″

To find arc 2.

Arc. 1	37°	58′	0″	cosec. .	0·210982
Pol. dist. at G. B. .	97	5	52	cosec. .	0·003339
Diff.	59	7	52		
Pol. dist. at L. B. .	97	8	32		
Sum	156	16	24	½ hav. .	4·990618
Diff.	38	0	40	½ hav. .	4·512779
				Hav. arc 2 . . .	9·717718
				Arc 2	92° 31′ 45″

To find arc 3.

Arc 1	37°	58′	0″	cosec. .	0·210982
Z. D. at G. B. . .	80	45	59	cosec. .	0·005664
Diff.	42	47	59		
Z. D. at L. B. . .	64	41	22		
Sum	107	29	21	½ hav. .	4·906540
Diff.	21	53	23	½ hav. .	4·278481
				Hav. arc 3 . . .	9·401667
				Arc 3	60° 17′ 0″

To find arc 4.

Arc 2	92° 31′ 45″
Arc 3	60 17 0
Arc 4	32 14 45
Pol. dist. at G. B.	97° 5′ 52″
Zen. dist. at G. B.	80 45 59
Diff.	16 19 53

To find arc 5.

Sin. pol. dist. at G. B.	9·996661
Sin. zen. dist. at G. B.	9·994336
Hav. arc 4	8·887148
Hav. arc 5	8·878145
Arc 5	31° 54′ 15″

Vers. 31° 54′ 15″	0151066
Vers. 16 19 53	0040349
Vers. colat	0191415
∴ Colat.	36° 2′ 32″
	90
Latitude	53 57 28 N.

(143.) June 3, 1847, in latitude by account 52° N., and long. 72° E., the following double altitude of the sun was observed.

Mean time nearly.	Chro.	Alt. sun's L. L.	Bearing.
9^h 50^m A.M.	9^h 52^m 28^s	51° 17′ 45″	S. E. b. S.
11 15 A.M.	11 14 29	59 32 15	S. b. E.

The run of the ship in the interval was W. by S. 10 miles, index correction — 0′ 40″ and height of eye 12 feet, required the true latitude at the second observation.

Ans., 50° 48′ N.

(144.) April 11, 1847, in latitude by account 50° 20′ N. long. 10° 30′ E., the following double altitude of the sun was observed.

Mean time nearly.	Chro.	Alt. sun's L. L.	Bearing.
11^h 0^m A.M.	7^h 5^m 10^s	40° 10′ 15″	S. S. E.
2 0 P.M.	10 6 10	35 15 40	S. W. b. W.

The run of the ship in the interval was N.N.E. 29 miles, index correction + 2′ 10″ and height of eye 18 feet; required the true latitude at the second observation.

Ans., 56° 56′ N.

(145.) April 13, 1847, in latitude by account 41° 20′ N.,

long. 156° 15′ E., the following double altitude of the sun was observed.

Mean time nearly.	Chro.	Alt. sun's L. L.	Bearing.
$10^h\ 45^m$ A.M.	$7^h\ 30^m\ 20^s$	53° 0′ 20″	S.E. b. S.
2 45 P.M.	11 29 40	40 59 10	S.W. b. W.

The run of the ship in the interval was S.S.E. 25 miles, index correction was — 5′ 20″ and height of eye 14 feet, required the true latitude at second observation.

Ans., 41° 23′ N.

(146.) April 22, 1847, in latitude by account 50° 48′ N., and long. 148° 30′ E., the following double altitude of the sun was observed.

Mean time nearly.	Chro.	Alt. sun's L. L.	Bearing.
$10^h\ 0^m$ A.M.	$10^h\ 2^m\ 25^s$	44° 20′ 0″	S. E. b. S.
11 24 A.M.	11 24 34	50 20 0	S. b. E.

The run of the ship in the interval was 0, index correction + 40″ and height of eye 0, required the true latitude at second observation. Ans., 50° 41′ N.

(147.) Oct. 15, 1848, in latitude by account 53° N., and long. 54° E., the following double altitude of the sun was observed.

Mean time nearly.	Chro.	Alt. sun's L. L.	Bearing.
$11^h\ 20^m$ A.M.	$11^h\ 15^m\ 50^s$	27° 31′ 50″	S. b. E.
1 20 P.M.	0 50 32	25 45 5	S. S. W.

The run of the ship in the interval was S. by W. 14 miles, index correction + 2′ 55″ and height of eye above the sea 15 feet, required the true latitude at second observation.

Ans., 53° 17′ N.

(148.) Oct. 24, 1849, in latitude by account 50° 40′ S., and long. 142° W., the following double altitude of the sun was observed.

Mean time nearly.	Chro.	Alt. sun's L. L.	Bearing.
$10^h\ 0^m$ A.M.	$10^h\ 12^m\ 34^s$	44° 20′ 0″	S. E. b. S.
11 24 A.M.	11 34 34	50 20 0	S. b. E.

The run of the ship in the interval was 0, and height of

eye above the sea 0, required the true latitude at second observation. Ans., 50° 45′ S.

Elements from Nautical Almanac.

Sun's declination.		Sun's semi.
June 2, 22° 8′ 50″ N.	June 3, 22° 16′ 34″ N.	15′ 47″
April 10, 7 49 12 N.	April 11, 8 11 21 N.	15 58
April 12, 8 33 22 N.	April 13, 8 55 14 N.	15 57
April 21, 11 44 26 N.	April 22, 12 4 46 N.	15 56
Oct. 14, 8 18 13 S.	Oct. 15, 8 40 28 S.	16 4
April 24, 11 49 32 S.	April 25, 12 10 19 S.	16 7

Second. When the objects observed are two stars taken at the same instant.

Rule XXXVII.

1. Correct the observed altitudes for index correction, dip, and refraction, and thus find the true altitudes, which subtract from 90° for the true zenith distances.

2. Take out of the Nautical Almanac the right ascension and declination of the two stars, and get their polar distances as in (3) p. 144.

3. *To find the polar angle.* The difference between the right ascensions of the two stars is the polar angle.

4. *To find arc* 1 (using Inman's Tables). Put down the two polar distances under each other, and take their difference. Add together the log. sin of the polar distance at greater bearing, the log. sin. of polar distance at less bearing, and the log. haversine of polar angle; the result, rejecting 10 in the index, is the log. haversine of an arc, which take from the tables and call arc A.

Add together versine of arc A and versine of the difference of polar distances; the sum will be the versine of arc 1, which find in the tables. Then proceed to find arc 2, &c., as in Rule 36, p. 145.

EXAMPLE.

January 1, 1846, in latitude by account 38° 10′ N.,

the following altitudes of the stars α Pegasi and α Aquilæ were taken at the same instant.

Obs. alt. α Pegasi.	Bearing.	Obs. alt. α Aquilæ.	Bearing.
29° 49′ 27″	E. b. S.	57° 29′ 50″	S. S. E.
In. cor. — 15″		In cor. — 15″	

The height of the eye was 41 feet, required the latitude.

At greater bearing. α Pegasi.				At less bearing. α Aquilæ.			
Observed alt. .	29°	49′	27″	Observed alt. .	57°	29′	50″
Index correction			15 —	Index correction			15 —
	29	49	12		57	29	35
Dip		6	18	Dip.		6	18 —
	29	42	54		57	23	17
Refraction . .		1	42 —	Refraction . .		0	37 —
True alt. . . .	29	41	12	True alt. . . .	57	22	40
	90				90		
Zenith distance .	60	18	48	Zenith distance .	32	37	20
Star's declination	14°	22′	50″ N.	Star's declination	8°	28′	2″N.
	90				90		
P. D. at G. B. .	75	37	10	Pol. dist. at L. B.	81	31	58
R. A. α Pegasi .	22ʰ	57ᵐ	6ˢ	Pol. dist. at G. B.	75°	37′	10″
R. A. α Aquilæ .	19	43	15	Pol. dist. at L. B.	81	31	58
Polar angle . .	3	13	51	Diff. pol. dists. .	5	54	48

To find arc 1.

Sin. polar distance at greater bearing . .	9·986177
Sin. polar distance at lesser bearing . .	9·995241
Haversine polar angle	9·226458
Haversine arc A	9·207876
Arc A	47° 22′ 30″
Vers. arc A	0322696
	107
Vers. difference polar distances	5319
Vers. arc 1	0328122
Arc 1	47° 47′ 16″

To find arc 2.

Arc 1	47°	47′	16″	. . . .	cosec. . .	0·130382
P. D. at G. B. . .	75	37	10	. . . .	cosec. . .	0·013823
Difference . . .	27	49	54			
P. D. at L. B. . .	81	31	58			
Sum (S)	109	21	52	. . . .	½ haversine	4·911662
Difference (D) . . .	53	42	4	. . . .	½ haversine	4·654808
					Haversine, arc 2 . .	9·710675
					Arc 2	91° 34′ 0″

To find arc 3.

Arc 1	47°	47′	16″	. . . .	cosec. . .	0·130382
Zen. dist. at G. B. .	60	18	48	. . . .	cosec. . .	0·061110
Difference . . .	12	31	32			
Zen. dist. at L. B. .	32	37	20			
Sum (S)	45	8	52	. . . .	½ haversine	4·584171
Difference (D) . .	20	5	48	. . . .	½ haversine	4·241725
					Haversine, arc 3 . .	9·017388
					∴ Arc 3	37° 38′ 30″

To find arc 4.

Arc 2	91°	34′	0″
Arc 3	37	38	30
Arc 4	53	55	30
P. D. at G. B. . .	75°	37′	10″
Z. D. at G. B. . .	60	18	48
Difference . . .	15	18	22

To find arc 5.

Sin. pol. dist. at G. B.	9·986177
Sin. Z. D. at G. B. . .	9·938890
Haversine, arc 4 . .	9·312977
Haversine, arc . . .	9·238044
Arc	49° 9′ 15″
Vers. arc.	0345919
	55
Vers. difference . . .	0035443
	28
Vers. arc 5	0381445
	363
	82
Arc 5 . . .	51° 47′ 22″
	90
Latitude . .	38 12 38 N.

(149.) Sept. 17, 1844, in latitude by account 36° 45′ N., the following altitudes were observed at the same time.

Obs. alt. α Orionis.	Bearing.	Obs. alt. α Leonis.	Bearing.
55° 1′ 30″	S.S.W.	45° 13′ 30″	S.E.

The index correction was + 55″ and height of eye was 8 feet, required the true latitude. Ans., 36° 44′ N.

(150.) Feb. 20, 1846, in latitude by account 36° 40′ N., the following altitudes were observed at the same time.

Obs. alt. Sirius.	Bearing.	Obs. alt. Spica.	Bearing.
27° 50′	S.W.	12° 56′	E.S.E.

The index correction was + 1′ and height of eye above the sea 10 feet, required the true latitude.

Ans., 36° 37′ N.

(151.) May 1, 1845, in latitude by account 41° 20 N. the following altitudes of stars were taken at the same instant, required the true latitude.

True alt. α Pegasi.	Bearing.	True alt. α Tauri.	Bearing.
62° 44′	S. b. E.	19° 26′ 20″	E.

Ans., 41° 22′ N.

(152.) March 2, 1845, in latitude by account 41° 20′ N. long., 60° E., the altitudes of the two following stars were observed at the same time, required the true latitude.

True alt. α Andromedæ.	Bearing.	True alt. α Tauri.	Bearing.
73° 14′	S. b. E.	18° 27′ 30″	E.

Ans., 41° 23′ N.

(153.) January 2, 1847, in latitude by account 38° 10′ N., the following altitudes of the stars α Pegasi and α Aquilæ were observed at the same instant.

Obs. alt. α Pegasi.	Bearing.	Obs. alt. α Aquilæ.	Bearing.
22° 49′ 27″	E. b. S.	57° 29′ 50″	S.S.E.

The index correction — 15″ and height of eye above the sea 41 feet, required the true latitude. Ans., 32° 43′ N.

(154.) Dec. 27, 1847, the following altitudes were observed

at the same instant, in latitude by account 37° 10′ N., required the true latitude.

True alt. β Orionis.	Bearing.	True alt. α Hydræ.	Bearing.
31° 5′ 11″	S.W. b. W.	39° 47′ 33″	S.E. ½ S.

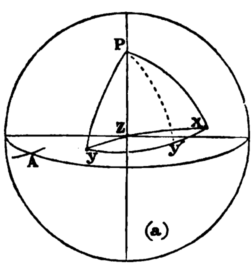
Ans., 37° 13′ N.

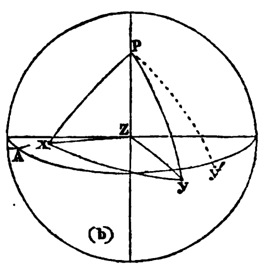
Elements from Nautical Almanac.

	Star's right ascension.				Star's declination.			
α Leonis	5^h	46^m	$47^s{\cdot}9$		12°	43′	24″	N.
α Orionis	10	0	5·8		7	22	23	N.
Spica	13	17	7·4		10	21	30	S.
Sirius	6	38	23·8		16	30	52	S.
α Tauri	4	27	3·0		16	11	30	N.
α Pegasi	22	57	4·0		14	22	24	N.
α Andromedæ	0	0	23·4		28	14	11	N.
α Pegasi	22	57	8·5		14	23	8	N.
α Aquilæ	19	43	18·2		8	28	12	N.
β Orionis	5	7	15·5		8	23	5	S.
α Hydræ	9	20	8·2		8	0	15	S.

Third. When the objects are two heavenly bodies observed at different times.

Let x, figs. (*a*) and (*b*), be the place of the heavenly body





first observed, and *y* the place of the second body when its altitude was taken, and let *y′* be the place of the second body when the first altitude was taken. Then in the elapsed

time (t) between the observations (which measures the angle y P y'), the second heavenly body has moved from y' to y, and the polar angle between the two observed places of the bodies, namely, X P y is equal, in fig. (a), to A P X $-$ A P y' $+ t =$ (A P X $+ t$) $-$ A P y', and, in fig. (b), to A P y' $-$ A P X $- t =$ A P y' $-$ (A P X $+ t$) (t the elapsed time being expressed in sidereal time). Hence the preceding rule for finding the latitude may be adapted to this case, using the angle X P y (the polar angle between the two places observed) instead of X P y' the difference of the right ascensions of the heavenly bodies.

The polar angle X P y may be found by the following

Rule XXXVIII.

1. Subtract the time shown by the chronometer at the first observation (increased if necessary by 12 hours) from the time shown at the second observation, and thus find the elapsed time.

2. * Correct the elapsed time for rate of chronometer, if any, either by proportional logs. or by the common rule of proportion.

3. Add to the elapsed time so corrected, the acceleration of sidereal on mean solar time (taken from table in Nautical Almanac or elsewhere). The result is the elapsed time expressed in sidereal time.

4. Add this elapsed time to the right ascension of the heavenly body first observed, and take the difference between the sum and the right ascension of the second heavenly body; the remainder (subtracted from 24 hours if greater than 12 hours) will be the polar angle required.

EXAMPLES.

1. The altitude of α Pegasi was observed when the chronometer showed $6^h\ 42^m\ 10^s$, and the altitude of

* When great accuracy is not required, and the elapsed time is small, these two corrections in 2 and 3 for rate of chronometer and acceleration may be omitted.

a Aquilæ was observed when the chronometer showed $8^h\ 32^m\ 5^s$, required the polar angle between the two places observed; the rate of the chronometer being $12^s{\cdot}5$ gaining.

Times by chronometer.

		h	m	s
At second observation		8	32	5
At first observation		6	42	10
		1	49	55
Gr. date log. sun for $1^h\ 49^m$.	1·11697			
Prop. log. for $12^s{\cdot}5$	2·93651			
	4·05348			1 —
		1	49	54
1^h . .	$9^s{\cdot}86$			
49^m .	8 ·05			
54^s. .	·15			
	18 ·06 .			18 +
Elapsed time in sidereal time		1	50	12
Right ascen. *a* Pegasi		22	57	14
		24	47	26
a Aquilæ		19	43	25
Polar angle required		5	4	1

2. The altitude of Sirius was observed when the chronometer showed $2^h\ 10^m\ 20^s$, and the altitude of Spica was observed when the chronometer showed $3^h\ 20^m\ 15^s$, required the polar angle between the two places observed; the rate of chronometer being $2^s{\cdot}5$ losing.

Times by chronometer.

		h	m	s
At second observation		3	20	15
At first observation		2	10	20
		1	9	55
Rate of chronometer				0
Acceleration . . . 1^h . . .	$9^s{\cdot}8$	1	9	55
9^m . . .	1 ·5			
55^s. . .	·1			
	11·4 .			11·4 +
		1	10	6·4
Right ascen. Sirius		6	38	25·4
		7	48	31·8
Spica		13	17	10·9
Polar angle		5	28	39·1

(155.) The altitude of β Orionis was observed when the chronometer showed $6^h\ 10^m\ 25^s$, and the altitude of α Hydræ was observed when the chronometer showed $7^h\ 17^m\ 35^s$, required the polar angle between the two places observed, the rate of chronometer being $6^s{\cdot}3$ losing, and the right ascension of β Orionis $5^h\ 7^m\ 15^s$, and of α Hydræ $9^h\ 20^m\ 8^s{\cdot}2$. Ans., $3^h\ 5^m\ 32^s$.

Rule XXXIX.

Given, the altitudes of two heavenly bodies observed at different times, to find the latitude.

1. Proceed as in (1) and (2) p. 150.
2. Find the polar angle as in Rule 38, p. 155.
3. Find arc 1, as in (4) p. 150.
4. Then proceed to find arcs (2) (3) (4) &c., as in Rule 36, p. 145.

EXAMPLE.

Sept. 27, 1846, in latitude by account 43° 30′ N., the following altitudes of the stars α Pegasi and α Aquilæ were observed at different times.

	Observed altitude.	Time by chron.	Bearing.
α Pegasi . .	29° 49′ 30″ .	$7^h\ 35^m\ 10^s$.	S.E.
α Aquilæ . .	54 29 0 .	8 2 10 .	S.¼W.

The run of the ship in the interval was S. 10 miles, the index correction + 1′ 10″ and height of eye above the sea 20 feet, required the true latitude at the second observation.

At greater bearing. α Pegasi.					At less bearing. α Aquilæ.				
Observed alt. . .	29°	49′	30		Observed alt. . .	54°	29′	0	
		1	10	+			1	10	+
	29	50	40			54	30	10	
		4	24	—			4	24	—
	29	46	16			54	25	46	
		1	41	—			0	41	—
	29	44	35			54	25	5	
		7	6	+	Zenith distance .	35	34	55	
	29	51	41						
Zenith distance .	60	8	19						

Right asc. α Pegasi, $22^h\ 57^m\ 9^s$.
Declination, 14° 23′ 9″ N.

Right asc. α Aquilæ, $19^h\ 43^m\ 19^s$.
Declination, 8° 28′ 9″ N.

To find the polar angle.

Chronom. showed at first observation .	7^h	35^m	10^s
„ second „ .	8	2	10
Elapsed time	0	27	0
Right ascension α Pegasi	22	57	9
	23	24	9
Right ascension α Aquilæ	19	43	19
Polar angle	3	40	50

To find arc 1.

Pol. dist. at G. B. . .	75° 36′ 51″
Pol. dist. at L. B. . .	81 31 41
Diff. pol. distances . .	5 54 50
Sin. pol. dist. at G. B. .	9·986161
Sin. pol. dist. at L. B. .	9·995236
Hav. pol. angle . . .	9·331838
Hav. arc A	9·313235
Arc A	53° 56′ 30″
Vers. arc A	0411274
	117
Vers. diff. pol. dists. .	0005297
	23
Vers. arc 1	0416711
Arc 1. . . 54° 19′ 4″	695
	16

To find arc 2.

Arc 1.	54° 19′ 4″
Pol. dist. at G. B. .	75 36 51
Diff.	21 17 47
Pol. dist. at L. B. .	81 31 41
Sum	102 49 28
Diff.	60 13 54
Cosec. arc. 1 . . .	090309
Cosec. pol. dis. at G. B.	013839
½ hav. sum	4·893016
½ hav. diff.	4·700498
Hav. arc 2	9·697662
Arc 2	89° 49′ 45″

To find arc 3.

Arc. 1	54° 19′ 4″
Zenith dist. at G. B. .	60 8 19
Diff.	5 49 15
Zenith dist. at L. B. .	35 34 55
Sum	41 24 10
Diff.	29 45 40
Cosec. arc 1	·090309
Cosec. zen. dis. at G. B.	·061869
½ hav. sum	4·548400
½ hav. diff.	4·409623
Hav. arc 3	9·110201
Arc 3	42° 4′ 45″

To find arc 4.

Arc 2	89° 49′ 45″
Arc 3	42 4 45
Arc 4	47 45 0

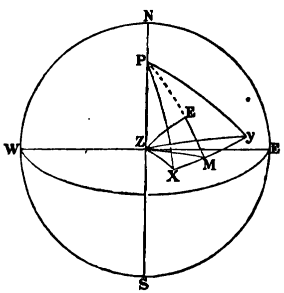
To find the latitude.

Log. sin. polar dist. at greater bearing .	9·986161
Log. sin. zenith dist. ,, .	9·938131
Log. hav. arc 4	9·214358
Hav. arc 5	9·138650
Arc 5	43° 33′ 0″
Vers. arc 5	0275227
Vers. diff. polar dist. and zenith dist. .	36214
	41
	0311482
	34
	48
	46° 29′ 14″
	90
Latitude . .	43 30 46 N.

Latitude by Ivory's rule for double altitude.

Let X and y be the place of the sun or a star at the times when its altitudes are taken.

Then we have given the polar distances Py, PX, the zenith distances Zy and ZX, and angle XPy to find the colatitude PZ, and thence the latitude.



Bisect Xy (an arc of a great circle passing through X and y) in M, join PM and ZM and draw ZE at right angles to PM. Then PM is at right angles to Xy, and ZMX is the complement of ZMP.

We have to compute the following arcs:

XM, MP, ZE, EM, and MP — EM = PE.

Then knowing ZE and PE in the right-angled triangle ZPE, we can find PZ the colatitude.

If the great circle drawn through X and Y pass when produced between the pole and the zenith, the perpendicular ZE will fall without the triangle PZM; in this case MP + EM = PE, and PE is formed by adding MP and EM together.

We may, however, determine whether the sum or difference is to be taken, by considering that since PZ must always be less than 90°, PE must likewise be less than 90°, and therefore if MP + EM exceeds 90°, we may be sure that PE = MP — EM or that PE is found by taking the difference between MP and EM.

The investigation from which the following rule is deduced will be found in the author's volume of "Astronomical Problems and Solutions."

Rule XL. (*Ivory's Rule.*)

1. From the time shown by the chronometer or watch at the second observation (increased if necessary by 12 hours) subtract the time shown at the first observation, divide by 2; the result is the half polar angle in time.

2. To the estimated mean time at the ship at the first observation add the half polar angle; the sum will be the ship mean time at the middle time between the observations.

3. Apply the longitude in time, and thus get a Greenwich date.

4. Take out from the Nautical Almanac the declination for this date, and also the sun's semidiameter in the adjacent column.

5. Correct the observed altitudes for index correction, dip, semidiameter, and parallax and refraction.

6. Correct also the first true altitude for run of ship in the interval, and thus get the true altitudes for the same place.

7. Put the first true altitude under the second true altitude, take their sum and difference, and also the half sum and half difference, call the half sum S. and the half difference D.

8. Under the log. sin. half polar angle put log. cos. declination: at the same time take out and put a little to the right, the log. sin. declination.

9. Add together the two logs. first taken out, and call the sum sin. arc 1.

10. At the same opening take out sec. arc 1, and put it under the log. sin. declination; take out also and put down in the same horizontal line the log. cosec. arc 1 and also log. sec. arc 1.

11. Add together log. sin. declination and log. sec. arc 1; the sum will be log. cos. arc 2; the arc corresponding thereto found in the Tables will be arc 2, if the latitude and

declination are of the same name, but if the latitude and declination are of different names, subtract the arc taken out from 180°, the remainder is arc 2.

12. Under log. cosec. 1, and log. sec. 1, just taken out, put the following quantities:—

Under log.	cosec. 1	put log.	cos. S.
„	sec. 1	„	sin. S.
„	cosec. 1	„	sin. D.
„	sec. 1	„	cos. D.

Add together log. cosec. 1 and the two logs. placed beneath it; the sum will be the log. sin. arc 3.

13. Take out the log. sec. arc 3, and put it down twice, once under log. cos. D, and again a little to the right.

14. Add together the log. sec. 1, and the three logarithms beneath it; the result is log. cos. arc 4, which find in the Tables.

15. Under arc 4 put arc 2, and take the difference in all cases when the line drawn through the places of the sun at the two observations will when produced *not* pass through the zenith and pole (that is, the difference must be taken, if it is seen that their sum would exceed 90°), otherwise take their sum; the result is arc 5.

Lastly. Under log. sec. arc 3, already taken out, put log. sec. arc 5; the sum will be the log. cosec. of the required latitude.

The arrangement on the paper of the logarithms to be taken out, as directed by the rule, will be better seen in the following blank form: and it would also facilitate the working out questions in other rules of Navigation if blank forms, similar to the one now given, were constructed on thick drawing paper by the student for each rule.

LATITUDE BY DOUBLE ALTITUDE OF SUN. (BLANK FORM.)

Times by Chronometer.

1st Obs.
Sub. from
* 2nd Obs.
Diff. or pol. A. .
½ Pol. A. . . . .

* Add 12 to hours at 2nd observation if necessary.

Sin. ½ pol. A.
+
Cos. decl.
Sin. (1) . . . . .

Mean T. at 1st observation.

Ship
+
½ pol. A.
Ship . . . . . .
Long. in T. . .
Gr. date . . . .
On . . . . . .

& Sin. decl.
+
Sec. (1)
Cos. (2) . . . .
* Arc (2) . . .
180
Arc (2) . . . .

* If latitude and declination of *different* names, subtract this arc from 180° for arc (2).

Sun's declination.

At
At
Diff. . . . . . .

Sun's decl. . . .

& Cosec. (1) . .
Cos. (S) . . . .
Sin. (D) . . . .
Sin. (3) . . . .

† If the sum of arc (4) and (2) equal or exceed 90°, then the *difference* is arc (5).

1st Observed altitude.

Obs. Alt. . . .
Index cor. . . .
Dip (—) . . .
Semidiameter .
Cor. in alt. (—)
Run . . . . .
1st Tr. alt. . . .

& Sec. (1) . . .
& Sin. (S) . . .
& Cos. (D) . .
& Sec. (3) . . .
Cos. (4) . . . .
Arc (4)
— or + †
Arc (2)
Arc (5) . . . .

2nd Observed altitude.

Obs. alt. . . .
Index cor. . . .
Dip (—) . . .
Semidiameter .
Cor. in alt. (—).
(2nd) Tr. alt. . .
(1st) Tr. alt. . .
Sum . . . . .
Difference . . .
(S) ½ Sum . . .
(D) ½ Difference

& Sec. (3)
+
Sec. (5)
Cosec. . . . .
Lat. . . . . .

EXAMPLES.

(156.) Oct. 11, 1845, in latitude by account 54° N., and long. 83° 15′ W., the following double altitude of the sun was observed.

Mean time nearly.	Chronometer.	Alt. sun's L. L.	Bearing.
7^h 45^m A.M.	11^h 40^m 15^s	9° 0′ 20″	E.S.E.
10 35 A.M.	2 13 20	25 3 30	S.S.E.

The run of the ship in the interval was S. by W. 15 miles, index correction + 5′ 10″, and height of eye above the sea was 18 feet, required the latitude at the second observation.

Ans., 53° 54′ N.

(157.) March 20, 1845, in latitude by account 52° 10′ N., and long. 55° 15′ W., the following double altitude of the sun was taken.

Mean time nearly.	Chronometer.	Alt. sun's L. L.	Bearing.
8^h 35^m A.M.	9^h 36^m	20° 0′ 30″	S.E.b.E.
1 45 P.M.	2 49	34 5 30	S.W.b.S.

The run of the ship in the interval was N.W. by W. 10 miles, index correction 0, and height of eye 20 feet, required the latitude at the second observation. Ans., 52° 27′ N.

(158.) Dec. 11, 1845, the following double altitude of the sun was observed.

Mean time nearly.	Chronometer.	Alt. sun's L. L.	Bearing.
6^h 0^m A.M.	6^h 3^m 30^s	19° 40′ 25′	E.b.S.
10 30 A.M.	10 4 25	50 20 40	N.E.

The run of the ship in the interval was E.N.E. 25 miles, index correction — 1′ 50″, and height of eye 16 feet, required the latitude at second observation, the latitude by account being 60° S. and long. 79° 15′ W.

Ans., 56° 59′ S.

(159.) Nov. 10, 1846, in latitude by account, 35° 30′ N., long. 94° 30′ E. the following double altitude of the sun was observed.

Mean time nearly.	Chronometer.	Alt. sun's L. L.	Bearing.
1^h 15^m P.M.	1^h 45^m 15^s	33° 5′ 40″	S.S.W.
3 45 P.M.	4 15 17	12 55 10	S.W.b.W.

The run in the interval was S.S.E. 15 miles, index correction + 4′ 10″, and height of eye 18 feet, required the true latitude at the second observation. Ans., 35° 31′ N.

(160.) Oct. 30, 1846, in latitude by account 52° 10′ N., and long. 159° 45′ E., the following double altitude of the sun was observed.

Mean time nearly.	Chronometer.	Alt. sun's L. L.	Bearing.
$11^h\ 15^m$ A.M.	$11^h\ 21^m\ 15^s$	25° 26′ 20″	S. ¾ E.
11 30 A.M.	11 37 55	25 55 0	S. ¼ E.

The run of the ship in the interval was S. by W. 1 mile, index correction + 3′ 50″ and height of eye above the sea was 20 feet, required the true latitude at second observation.

Ans., 49° 56′ N.

(161.) March 5, 1846, in latitude by account 60° N., and long. 46° W., the following double altitude of the sun was observed.

Mean time nearly.	Chronometer.	Alt. sun's L. L.	Bearing.
$10^h\ 10^m$ A.M.	$10^h\ 10^m\ 5^s$	19° 30′ 40″	S.S.E.
3 10 P.M.	3 10 40	15 2 30	S.W.

The run of the ship in the interval was S.W. by W. 15 miles, index correction + 2′ 10″ and height of eye 20 feet, required the true latitude at second observation.

Ans., 59° 59′ N.

Elements from Nautical Almanac.

	Sun's declination.		Sun's semi.
Oct. 11	7° 4′ 38″ S.	12 . . 7° 27′ 15″ S.	16′ 3″
Mar. 20	0 5 40 S.	21 . . 0 18 1 N.	16 4
Dec. 11	23 1 55 S.	12 . . 23 6 54 S.	16 16
Nov. 9	16 51 6 S.	10 . . 17 8 9 S.	16 11
Oct. 29	13 26 6 S.	30 . . 13 45 56 S.	16 8
Mar. 5	6 4 45 S.	6 . . 5 41 33 S.	16 8

A valuable extension of this problem has recently been made by Mr. Riddle, the head master of the Greenwich schools. It consists in finding the hour angle Z P M with very little additional labour, and thence apparent time at

the ship. For since in the triangle Z P E, sin. h = sin. arc 3, sec. lat., we have only to add to sin. arc 3, already taken out of the table, the log. sec. lat. to determine the hour angle h, which will also be ship-apparent time, if P M or what it wants of 24 hours if A.M.; by applying the equation of time we obtain mean time at the ship. If therefore we know, by means of the chronometer, mean time at Greenwich, at the same instant, we can readily find the longitude in time by the following rule.

Rule XLI.

Rule for finding the longitude by means of the observations of the sun for latitude by double altitude.

1. Find the equation of time for the Greenwich date.

2. To the log. sec. lat. add log. sin. arc 3 already known, the sum will be log. sin. hour angle at the middle time between the observations.

3. If P.M. at ship at the middle time, this will also be ship apparent time. If A.M., subtract the hour angle from 24 hours, the remainder is ship apparent time.

4. Apply the equation of time with its proper sign, and thus get ship mean time.

5. To the mean time shown by chronometer at the middle time between the observations (found by taking half the sum of the times by chronometer at first and second observations), apply the error of chronometer, and thus get Greenwich mean time.

6. The difference between Greenwich mean time and ship mean time is the long. in time. If the Greenwich time is the least, the longitude is east, otherwise west.

CHAPTER VII.

RULES FOR FINDING THE ERROR AND RATE OF CHRONOMETERS, BY SINGLE ALTITUDES AND BY EQUAL ALTITUDES.

To find the error and rate of chronometers.

THERE are two methods of determining the error of a chronometer on mean time, the one by a single altitude of a heavenly body observed at some distance from the meridian, the other by means of equal altitudes of a heavenly body observed on both sides of the meridian.

The mean daily rate of a chronometer is found by dividing the increase or decrease in its error by the number of days elapsed between the times when the observations were taken to determine its error; thus, suppose on April 27, at $9^h\ 30^m$ A.M., the error of a chronometer was found to be fast $10^m\ 10^s{\cdot}5$ on Greenwich mean time, and that on April 30th about the same hour its error was found to be $10^m\ 40^s{\cdot}5$ fast: then it appears that in the three days elapsed between the observations the chronometer has gained 30^s, hence its mean daily rate is 10^s gaining.

Before going to sea, the error of the chronometer on Greenwich mean time, and its daily rate, are supposed to have been accurately determined, either at an observatory by means of daily comparisons with an astronomical clock, or by observations taken with a sextant at a place whose longitude is known.

When the error and rate of a chronometer are given we may determine what its error will be on some future day, provided the rate of the chronometer continues uniform in the interval, by the following rule.

Rule XLII.

Given, the error of a chronometer on Greenwich mean time, and also its daily rate, to find Greenwich mean time at some other instant, as when an observation is taken, &c.

1. Get a Greenwich date.

2. Find the number of days and part of a day that have elapsed from the time when the error and rate were determined by the hour of the Greenwich date.

3. Multiply the rate of the chronometer by the number of days elapsed, and add thereto the proportionate part for the fraction of a day, found by proportion or otherwise. The result is the accumulated error in the interval.

4. If the chronometer is gaining, subtract the accumulated error from the time shown by the chronometer; if losing, add.

5. To the result apply the original error of chronometer, adding if slow, subtracting if fast (increasing the time shown by chronometer by 24^h if necessary, and putting the day one back). The result, (rejecting 24^h if greater than 24^h and putting the day one forward), will be mean time at Greenwich at the instant of the observation.

NOTE.—If this time differs from the Greenwich date by 12 hours nearly; in that case 12 hours must be added to the Greenwich time, determined as above, to get the astronomical Greenwich mean time.

EXAMPLES.

1. June 13, 1851, at 10^h 52^m P.M., mean time nearly, in long. 60° W., an observation was taken when a chronometer showed 2^h 50^m 42^s. On June 1, its error was known to be 3^m $10^s{\cdot}2$ fast on Greenwich mean time, and its mean daily rate was $3^s{\cdot}5$ gaining, required mean time at Greenwich when the observation was taken.

Ship, June 13	10^h	52^m	
Long. in time	4	0	W.
Greenwich, June 13 . .	14	52	

Interval from June 1 to June 13, at $14^h\ 52^m$ is $12^d\ 14^h\ 52^m = 12^d\ 15^h$ nearly

Daily rate . .			$3^s\cdot5$
			12
12^h is $\frac{1}{2}$			42 ·0
3 „ $\frac{1}{4}$			1 ·75
			·44
Accumulated rate			44 ·2 gaining
Chronometer showed . . .	2^h	50^m	$42^s\cdot0$
	2	49	57 ·8
Original error		3	10 ·2 fast
Greenwich, June 14 . . .	2	46	47 ·6 A.M.
∴ Greenwich mean time. .	14	46	47 ·6 when observation was taken.

2. Aug. 10, 1853, at $3^h\ 42^m$ A.M., mean time nearly in long. 100° 30′ W., an observation was taken when a chronometer showed $10^h\ 30^m\ 45^s\cdot5$.

On Aug. 1, its error was known to be $12^m\ 10^s\cdot5$ slow on Greenwich mean time and its rate $11^s\cdot2$ gaining, required mean time at Greenwich when the observation was taken.

Ship, August 9 . . .	15^h	42^m	
Long. in time . . .	6	42	W.
Greenwich, August 9 .	22	24	

Interval from Aug. 1 to Aug. 9, at $22^h\ 24^m$ is $8^d\ 22^h\ 24^m$

Daily rate			$11^s\cdot2$
			8
12^h is $\frac{1}{2}$ . . .			89 ·6
8 „ $\frac{1}{3}$ . . .			5 ·6
2 „ $\frac{1}{4}$ . . .			3 ·7
24^m „ $\frac{1}{4}$ nearly			·9
			·2
			100 ·0
Accumulated error . . .		1^m	40 ·0 gaining
Chronometer showed. . .	10^h	30	45 ·5
	10	29	5 ·5
Original error		12	10 ·5 slow
Greenwich, August 10 . .	10	41	16 ·0 A.M.
Or, Greenwich, August 9 .	22	41	16 ·0

If the Greenwich time thus determined differs considerably from the Greenwich date used, the work should be repeated, using for the Greenwich date the approximate Greenwich time first found.

EXAMPLES.

(162.) Nov. 20, 1851, at $6^h\ 42^m$ P.M., mean time nearly, in long. 32° 0′ E., an observation was taken when a chronometer showed $4^h\ 30^m\ 6^s$.

On Oct. 9, its error was known to be $5^m\ 52^s\cdot4$ slow on Greenwich mean time, and its rate $2^s\cdot7$ losing: required mean time at Greenwich when the observation was taken.

Ans., $4^h\ 36^m\ 52^s\cdot3$.

(163.) Dec. 31, 1851, at $10^h\ 10^m$ A.M. mean time nearly, in long. 150° E., an observation was taken when a chronometer showed $0^h\ 0^m\ 22^s\cdot3$.

On Nov. 20, its error was known to be $3^m\ 52^s\cdot4$ slow on Greenwich mean time, and its rate $2^s\cdot7$ losing: required mean time at Greenwich when the observation was taken.

Ans., $12^h\ 6^m\ 4^s\cdot0$.

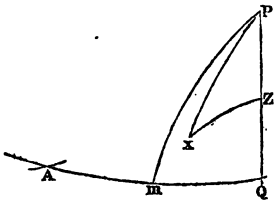
(164.) April 11, 1851, at $3^h\ 14^m$ P.M. mean time nearly, in long. 56° 42′ W., an observation was taken, when a chronometer showed $7^h\ 2^m\ 10^s\cdot5$.

On March 15, its error was known to be $1^m\ 32^s\cdot7$ fast, on Greenwich mean time, and its daily rate $6^s\cdot3$ losing: required mean time at Greenwich, when the observation was taken. Ans., $7^h\ 3^m\ 29^s\cdot7$.

To find the error of a chronometer on mean time at the place of observation, by a single altitude of the sun.

Let P be the pole, Z the zenith, and X the place of the sun, bearing as nearly east or west as possible, A Q the celestial equator, A the first point of Aries, and m the place of the mean sun. Then in the triangle Z P X are given the zenith distance Z X, the polar distance P X, and the colatitude of the

spectator P Z, to find the hour angle Z P X, which is also the apparent solar time, if the sun is west of the meridian, or what it wants of 24 hours if the sun is east of meridian. To this apparent solar time thus found, apply the equation of time X P m with its proper sign, as given in the Nautical Almanac, the result will be Q P m, or mean time at the place of observation: the difference between which and the time shown by the chronometer at the instant of the observation will manifestly be the error of the chronometer on mean time at the place. Hence this rule.



Rule XLIII.

1. Find a Greenwich date.

2. Correct the sun's declination and equation of time for this date. Take out of the Nautical Almanac the sun's semidiameter, at the same time the declination and equation of time are taken out.

3. Correct the observed altitude for index correction, dip, semidiameter, and correction in altitude, and thus get the true altitude; subtract the true altitude from 90° to obtain the zenith distance.

4. *To find ship apparent time.* (using log. haversines*).

* If the student have no table of haversines, he may proceed as follows to find apparent solar time :—

Under the latitude put the sun's declination, and, if the names be alike, take the difference; but if unlike, take their sum. Under the result put the zenith distance, and find their sum and difference, and half-sum and half-difference.

Add together the log. secants of the two first terms in this form (rejecting the tens in index) and the log. sines of the two last, and divide the sum by 2; look out the result as a log. sine and multiply the angle taken out by 2.

Under the latitude put the sun's declination, and, if the names be alike, take the difference; but if unlike, take their sum. Under the result put the zenith distance, and find their sum and difference. Add together the log. secants of the two first terms in this form, and the halves of the log. haversines of the two last; and (rejecting the tens in the index) look out the sum as a log. haversine, to be taken out at the top of the page if the sun is west of the meridian, but at the bottom of the page if the sun is east of meridian. The result is apparent solar time at the instant of observation.

5. *To find mean time.* To apparent solar time apply the equation of time with its proper sign, as directed in the Nautical Almanac; the result is mean time at the place.

6. The difference between mean time thus found, and the time shown by chronometer at the observation, will be the error of the chronometer on mean time at the place.*

Rule XLIV.

To find the error of a chronometer on mean time at Greenwich by a single altitude of the sun.

Find mean time at the place of observation as directed in preceding Rule. See 1, 2, 3, 4, and 5.

6. To the mean time at the place thus found apply the longitude in time; adding if west, and subtracting if east (rejecting or adding 24 hours if necessary): the result will be mean time at Greenwich at the time of the observation.

Reduce the angle thus found into time, and if the sun is west of meridian, the same will be apparent time; but if east of meridian, subtract the angle from 24 hours; the remainder will then be apparent solar time at the instant of observation.

* A similar observation being taken a few days afterwards, the *mean daily rate* may be found as pointed out in p. 167.

7. The difference between which and the time shown by chronometer will be the error of the chronometer on Greenwich mean time.

EXAMPLE.

May 10, 1842, at $8^h\ 44^m$ A.M., mean time nearly, in latitude 50° 48′ N., and long. 1° 6′ W., when a chronometer showed $8^h\ 26^m\ 59^s{\cdot}7$, the observed altitude of the sun's lower limb was 39° 14′ 30″, index correction + 4′ 24″ and height of eye above the sea 20 feet, required the error of the chronometer on mean time at the place, and also its error on Greenwich mean time.

Ship, May 9	20^h	44^m
Long. in time . . .		4 W.
Greenwich, May 9 . .	20	48

Sun's declination.		Equation of time.		Sun's semi.
9th . . .	17° 19′ 24″N.	9th . . .	$3^m\ 46^s{\cdot}2$ sub.	15′ 51″
10th . . .	17 35 18 N.	10th . . .	3 49 ·1	
	15 54		2 ·9	
·06215		·06215		
1·05388		3·57103		
1·11603	13 46	3·63318	2 ·5	
Declination	17 33 10 N.		3 48 ·7	

Observed altitude	39°	14′	30″	
Index correction.		4	24	+
	39	18	54	
Dip		4	24	—
	39	14	30	
Semidiameter.		15	51	+
	39	30	21	
Correction in altitude . . .		1	3	—
	39	29	18	
	90			
Zenith distance	50	30	42	

To find ship apparent time (using haversines).

Latitude	50° 48′ 0″ N.		Sec. . .	0·199263
Declination . . .	17 33 10 N.		Sec. . .	0·020710
Difference	33 14 50			
Zenith distance . .	50 30 42			
Sum	83 45 32	. . . . .	½ haver. .	4·824491
Difference	17 15 52	. . . . .	½ haver. .	4·176307
	Hav. of angle			9·220771

∴ Ship apparent time . . .	20^h	47^m	30^s
Equation of time		3	48 ·7
Ship mean time.	20	43	41 ·3
Chronometer showed	20	26	59 ·7
Error of chronom. on mean time at place		16	41 ·6 slow.

To find error on Greenwich mean time.

Ship mean time	20^h	43^m	$41^s·3$
Longitude in time		4	24 ·0 W.
Greenwich mean time	20	48	5 ·3
Chronometer showed	20	26	59 ·7
Error of chro. on Gr. mean time . .		21	5 ·6

To find ship apparent time (using the common tables of log. sines, &c. Note, p. 171).

Latitude	50° 48′ 0″ N.	. . .	Sec. . .	0·199263
Declination . . .	17 33 10 N.	. . .	Sec. . .	0·020710
	33 14 50			
Zenith distance . .	50 30 42			
Sum.	83 45 32			
Difference.	17 15 52			
½ Sum	41 52 46	. . . .	Sin. . .	9·824491
½ Difference . . .	8 37 56	. . . .	Sin. . .	9·176300
				2) 19·220764
			Sin. . .	9·610382
				1^h 36^m 15^s
				2
				3 12 30
				24
	Ship apparent time . . .			20 47 30

If the computed ship mean time differ several minutes from the estimated ship mean time, it will be advisable, when great accuracy is required, to recalculate the sun's declination and the hour angle; using the approximate ship time just found to determine the Greenwich date; the following example will illustrate the mode of proceeding:—

March 16, 1844, at $10^h\ 10^m$ A.M, mean time nearly, in lat. 50° 48′ N., and long. 1° 6′ W., when a chronometer showed $10^h\ 15^m\ 47^s{\cdot}2$, the observed altitude of the sun's lower limb was 58° 46′ 30″ (in artificial horizon), the index correction + 1′ 20″, required the error of chronometer on Greenwich mean time.

March 15	22^h	10^m
Long. in time . . .		4 W.
Greenwich, March 15 .	22	14

Sun's declination.		Equation of time.		Sun's semi.
15th . . .	1° 58′ 28″ S.	15th . . .	$9^m\ 1^s{\cdot}7$ add	16′ 5″
16th . . .	1 34 45 S.	16th . . .	8 44·4	
	23 43		17·3	
·03321		·03321		
·88022		2·79538		
·91343	21 58	2·82859	16·0	
Declination	1 36 30 S.		8 45·7	

	°	′	″	
Observed altitude	58	46	30	
Index correction		1	20	+
2)	58	47	50	
	29	23	55	
Semidiameter		16	5	
	29	40	0	
Correction in altitude		1	34	—
	29	38	26	
	90			
Zenith distance	60	21	34	

Latitude	50° 48′ 0″ N. . . .	Sec. . .	0·199263
Declination . . .	1 36 30 S. . . .	Sec. . .	0·000171
Sum.	52 24 30		
Zenith distance . .	60 21 34		
Sum.	112 46 4	½ hav. .	4·920520
Difference. . . .	7 57 4	½ hav. .	3·840866
		Hav. . .	8·960820

Apparent time	21^h	39^m	14^s
Equation of time . . .		8	45·7 +
Mean time.	21	47	59·7
Long. in time		4	24·0 W.
Greenwich mean time. .	21	52	23·7
Chronometer showed . .	22	15	47·2
Error of chronometer on Greenwich mean time .		23	23·5

The mean time at the place is found to be $21^h\ 47^m\ 59^s{\cdot}7$, but the mean time used for computing the declination and equation of time was $22^h\ 10^m$. Now this has rendered the declination slightly incorrect, and therefore the time computed from it. When it is desirable to obtain mean time at the place as correctly as possible, we must recalculate the declination and apparent time, using the approximate mean time for finding a more correct Greenwich date; thus the mean time at the place is found above to be $21^h\ 47^m\ 52^s{\cdot}7$, assuming therefore the mean time to be $21^h\ 48^m$, obtain a second Greenwich date, and recompute the sun's declination and hour angle as follows:—

March 15, mean time . .	21^h	48^m
Long. in time		4 W.
Greenwich, March 15 . .	21	52

	Sun's declination.					
15th	1° 58′ 28″ S.					
16th	1 34 45 S.					
	23 43					
·04043						
·88022						
·92065	21 36					
Declination . . .	1 36 52 S.	. . .	Sec. . .	0·000171		
Latitude	50 48 0 N.	. . .	Sec. . .	0·199263		
	52 24 52					
Zenith distance .	60 21 34					
Sum	112 46 26	. . . .	½ hav. . .	4·920540		
Difference . . .	7 56 42	. . . .	½ hav. . .	3·840630		
				8·960605		

Apparent time . . $21^h\ 39^m\ 17^s$

Whence the error of chronometer is fast $23^m\ 20^s{\cdot}5$ on Greenwich mean time.

(165.) May 20, 1847, at $5^h\ 20^m$ P.M., mean time nearly, in lat. 47° 20′ N., and long. 94° 30′ E., when a chronometer showed $11^h\ 5^m\ 20^s$, the observed altitude of the sun's lower limb was 20° 0′ 15″, the index correction — 4′ 10″ and height of eye above the sea 20 feet, required the error of chronometer on Greenwich mean time.

Ans., fast $0^m\ 41^s{\cdot}4$.

(166.) Feb. 3, 1847, at $10^h\ 30^m$ A.M., mean time nearly, in lat. 49° 30′ N., and long. 22° W., when a chronometer showed $0^h\ 2^m\ 30^s$, the observed altitude of the sun's lower limb was 19° 21′ 30″ the index correction + 3′ 20″, and height of eye above the sea 18 feet, required the error of chronometer on Greenwich mean time.

Ans., fast $10^m\ 7^s{\cdot}4$.

(167.) March 25, 1847, at $3^h\ 20^m$ P.M., mean time nearly, in lat. 52° 10′ N., and long. 36° 58′ 15″ W., when a chronometer showed $5^h\ 40^m\ 58^s$, the observed altitude of the sun's lower limb was 25° 10′ 20″, the index correction — 6′ 10″,

and height of eye above the sea 20 feet, required the error of chronometer on Greenwich mean time.

Ans., $9^m\ 25^s{\cdot}2$ slow.

(168.) May 19, 1847, at $3^h\ 0^m$ P.M., mean time nearly, in lat. 49° 50′ N., and long. 21° 4′ 45″ E., when a chronometer showed $1^h\ 23^m\ 20^s$, the observed altitude of the sun's lower limb was 42° 50′ 30″, the index correction + 4′ 10″, and height of eye above the sea 20 feet, required the error of chronometer on Greenwich mean time.

Ans., $10^m\ 37^s{\cdot}6$ slow.

Elements from Nautical Almanac.

	Sun's declination.	Equation of time.	Semi.
May 19	19° 41′ 37″ N.	$3^m\ 49^s{\cdot}6$ sub.	15′ 49″
„ 20	19 54 26 N.	3 46·2	
Feb. 2	16 54 7 S.	13 58·8 add.	16 14
„ 3	16 36 41 S.	14 5·6	
March 25	1 40 56 N.	6 13·8 add.	16 3
„ 26	2 4 29 N.	5 55·2	
May 19	19 41 37 N.	3 49·5 sub.	15 49
„ 20	19 54 26 N.	3 46·9	

Second. When the object observed is a star.

Rule XLV.

To find the error of chronometer on mean time at a place by a single altitude of a star.

1. Get a Greenwich date.

2. Take out of the Nautical Almanac the right ascension and declination of the star, and also the right ascension of the mean sun for mean noon of the Greenwich date.

3. Correct the right ascension of mean sun for Greenwich date (p. 83).

4. Correct the observed altitude for index correction, dip,

and refraction, and thus get the true altitude, which subtract from 90° for the true zenith distance.

5. *To find star's hour angle* (using haversines*). Under the latitude put the star's declination; add if the names be unlike, subtract if like; under the result put the true zenith distance of star, and take the sum and difference. Add together the log. secants of the two first terms in this form (omitting the tens in each index), and halves the log. haversines of the two last; the sum, (rejecting 10 in the index,) will be the log. haversine of hour angle, to be taken out at top of page if heavenly body be west of meridian, but at bottom if east of meridian.

6. To the hour angle thus found add the star's right ascension, and from the sum (increased if necessary by 24 hours) subtract the right ascension of mean sun; the remainder is mean time at the place at the instant of observation.

7. Under mean time at place put the time shown by chronometer; the difference will be the error of chronometer on mean time at place.

To find the error of chronometer on Greenwich *mean time.*

Proceed as in the corresponding rule for the sun, p. 172.

EXAMPLE.

June 3, 1842, at $12^h\ 9^m$ P.M., mean time nearly, in lat. 50° 48′ N., and long. 1° 6′ 3″ W., observed the altitude of α Bootis (west of meridian) to be 89° 53′ 30″ in artificial horizon, when a chronometer showed $0^h\ 14^m\ 22^s{\cdot}3$, the index correction was — 10″, required the error of the chronometer on mean time at the place, and also on Greenwich mean time.

* If the student have the table of log. sines, &c., only, the hour angle may be found in a similar manner as in note, p. 171, and example, p. 174.

June 3	12^h 9^m
Long. in time	4 +
Greenwich, June 3 . . .	12 13

Observed altitude	89°	53′	30″	
Index correction			10	—
	2) 89	53	20	
	44	56	40	
Refraction			58	—
	44	55	42	
	90			
True zenith distance . . .	45	4	18	

Star's right ascension . . .	14^h 8^m $30^s{\cdot}5$
Star's declination	20° 0′ 15″ N.

Right ascension of mean sun . . .	4^h	46^m	$7^s{\cdot}1$
Correction for 12^h		1	58·3
13^m			2·1
Right ascen. of mean sun at 12^h 13^m .	4	48	7·5

Latitude	50°	48′	0″	N. . .	Sec.	0·199263
Declination . . .	20	0	15	N. . .	Sec.	0·02702
	30	47	45			
Zenith distance . .	45	4	18			
Sum.	75	52	3	. . . .	½ hav. . . .	4·788699
Difference	14	16	33	. . . .	½ hav. . . .	4·094305
					Hav. hour ang.	9·109293

Hour angle	2^h	48^m	8^s
Star's right ascension . .	14	8	30·5
	16	56	38·5
Right ascen. mean sun. .	4	48	7·5
Mean time at place. . .	12	8	31·0
Chronometer showed . .	12	4	22·3
Error of chronometer, fast		5	51·3

To find the error of chronometer on Greenwich mean time.

Mean time at place	12^h	8^m	$31^s{\cdot}0$	
Long. in time		4	24·2	W.
Greenwich mean time	12	12	55·2	
Chronometer showed	12	14	22·5	
Error of chron. on Gr. mean time		1	27·1	fast.

(169.) May 4, 1847, at 4^h 40^m A.M., mean time nearly, in lat. 40° 10′ 20″ N., and long. 81° 47′ 15″ E., when a chronometer showed 11^h 13^m 50^s, the observed altitude of α Bootis (west of meridian) was 20° 45′ 4″·5, the index correction was — 2′ 10″, and the height of eye above the sea was 18 feet, required the error of the chronometer on Greenwich mean time. Ans., 0^m $35^s{\cdot}3$ slow.

(170.) Feb. 10, 1847, at 9^h 22^m P.M., mean time nearly, in lat. 28° 30′ N., and long. 27° 15′ W., a chronometer showed 11^h 17^m 20^s, when the observed altitude of α Leonis (east of meridian) was 42° 10′ 0″, the index correction — 3′ 20″, and height of eye above the sea 20 feet, required the error of the chronometer on Greenwich mean time.

Ans., 4^m $57^s{\cdot}3$ fast.

(171.) April 18, 1848, at 0^h 40^m A.M., mean time nearly, in lat. 46° 32′ N., and long. 43° 36′ 15″ E., when a chronometer showed 10^h 13^m 45^s, the observed altitude of the star α Aquilæ was 14° 45′ 15″ (east of meridian) the index correction + 4′ 5″, and height of eye above the sea 18 feet, required the error of the chronometer on Greenwich mean time. Ans., 19^m $31^s{\cdot}7$ fast.

(172.) Aug. 11, 1848, at 8^h 10^m P.M., mean time nearly, in lat. 50° 20′ N., and long. 29° 53′ 15″ E., when a chronometer showed 6^h 6^m $20^s{\cdot}0$, the observed altitude of α Bootis (Arcturus) was 39° 5′ 10″ (west of meridian) the index correction — 2′ 10″, and height of eye above the sea 18 feet, required the error of the chronometer on Greenwich mean time. Ans., 11^m $17^s{\cdot}8$ slow.

Elements from Nautical Almanac.

Right ascen. mean sun.			Right ascen. and decl. of star.		
May 3	$2^h\ 43^m\ 3^s{\cdot}3$	. . α Bootis	$14^h\ 8^m\ 43^s{\cdot}4$	$19°\ 58'\ 46''$	N.
Feb. 10	21 19 46·0	. . α Leonis	10 0 15·3	12 42 30	N.
April 17	1 42 57·3	. . α Aquilæ	19 43 22·7	8 28 16	N.
Aug. 11	9 20 17·8	. . α Bootis	14 8 45·0	19 58 39	N.

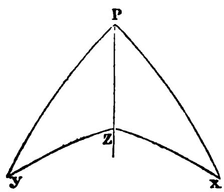
To find the error of a chronometer on mean time at a place by EQUAL ALTITUDES *of the sun.*

When the sun's centre is on the meridian of any place, the apparent time is then either 0^h or 24^h. To obtain mean time at the same instant, we have only to apply the equation of time with its proper sign. We thus find mean time at the instant the sun is on the meridian; and if we can also ascertain what a chronometer showed at the same instant, it is manifest that the error of the chronometer on mean time at the place is known, since it will be the difference between the two times.

To find the time shown by the chronometer at apparent noon, we have recourse to the method of equal altitudes, which consists in noting the time shown by the chronometer when the heavenly body has the same altitude on both sides of the meridian: half the interval between the observations being added to what the chronometer showed at the first observation will be the time shown by the chronometer when the heavenly body is on the meridian, if *the declination is supposed to be invariable* in the interval between the observations.

For let t and $t_{,}$ be the times shown by the chronometer when the heavenly body is at X and y, at the same altitude on both sides of P Z the meridian; and suppose $t_{,}$ greater than t (that is, if the hour hand has arrived in the interval to 12^h, we continue to count 13^h, 14^h, &c., instead of 1^h, 2^h, &c.). Now, if the rate of chronometer has been uniform

in the interval, the time elapsed is $t_{,} - t$, and the heavenly body has described the angle Z P X, or half y P X in the time $\frac{1}{2}(t_{,} - t)$. To this half interval add the time t shown by the chronometer when the body was at X, we find that the instant that the body arrived at the meridian the chronometer must have showed $t + \frac{1}{2}t_{,} - \frac{1}{2}t = \frac{1}{2}(t_{,} + t)$. The difference between $\frac{1}{2}(t_{,} + t)$ and mean time at noon will be the error of the chronometer on mean time at the place. For, let us suppose that the equation of time at noon is $4^m\ 24^s$ subtracted from apparent time; then mean time at the place when the sun is on the meridian is $24^h - 4^m\ 24^s = 23^h\ 55^m\ 36^s$. Now, if $\frac{1}{2}(t_{,} + t)$ is found to be $23^h\ 10^m\ 26^s$ the difference $45^m\ 10^s$ is the error of the chronometer slow on mean time at the place.



But the sun's declination is not invariable during the interval $t_{,} - t$, but increases or decreases by a small quantity, so that the angle ZPX differs from half the interval by a few seconds.

The following rule enables us to find the number of seconds which must be applied to the half interval to obtain Z P X. This quantity of time is called *the equation of equal altitudes.*

Rule XLVI.

To find the error of a chronometer on mean time by equal altitudes of the sun.

1. Find mean time nearly of apparent noon at the place by taking out of the Nautical Almanac the equation of time to the nearest minute, and applying it with its proper sign to 0^h or 24^h, according as the Nautical Almanac directs it to be added to or subtracted from apparent time, putting the day one back in the latter case.

2. To mean time nearly thus found apply the longitude in time, adding if west, and subtracting if east; the result will be a Greenwich date.

3. Correct the equation of time for this date.

4. From the P.M. time when the second altitude was taken (increased by 12 hours) subtract the A.M. time when the first altitude was taken; the remainder is elapsed time as shown by the chronometer: take half the elapsed time and subtract it from the above date (increased if necessary by 24 hours and the day put one back), the remainder is a second Greenwich date.

5. Take out the sun's declination for this date.

6. *To find the equation of equal altitudes.* Under heads (1) and (2) put down the following quantities.

Under (1) put A taken from annexed table.
„ (2) put B „
„ (1) put log. cotangent latitude.
„ (2) put log. cotangent declination.
„ both (1) and (2) put proportional log. change of declination in 24 hours.

7. Add together logarithms under (1) and (2) and reject the tens in the index; look out the result as a proportional logarithm, and take out the seconds and tenths corresponding thereto.

8. Mark the quantities under (1) plus (+) if the declination is decreasing, and of the same name as the latitude; or, if increasing and of a different name. Otherwise mark the quantity minus (—).

9. Mark the quantity under (2) plus (+) if the declination is increasing, but minus (—) if decreasing.

10. Take the sum or difference of these quantities, according as they have the same or different signs; the result will be the correction or equation of equal altitudes required.

11. Add together A.M. time and half elapsed time, and to the same apply the correction just found with its proper

sign: the result will be the time shown by the chronometer when the sun's centre is on the meridian.

12. Find mean time at the same instant by applying the equation of time to 0^h or 24^h with the proper sign as directed in the Nautical Almanac.

13. Put down under each other the results determined in (11) and (12), and take the difference, which will be the error of the chronometer on mean time *at the place.*

14. *To find error of the chronometer on Greenwich mean time.* To mean time at the place as found in (12) apply the longitude in time, and thus get mean time at Greenwich, under which put the time shown by chronometer as found in (11); the difference will be the error of the chronometer on *Greenwich* mean time.

EQUATION OF EQUAL ALTITUDES.

Elapsed time.		A	B	Elapsed time.		A	B	Elapsed time.		A	B
1	30	1·97148	1·97991	4	30	1·94886	2·02901	7	30	1·90212	2·15738
1	40	1·97082	1·98123	4	40	1·94692	2·03356	7	40	1·89876	2·16854
1	50	1·97009	1·98272	4	50	1·94490	2·03833	7	50	1·89531	2·18033
2	0	1·96930	1·98435	5	0	1·94281	2·04334	8	0	1·89177	2·19280
2	10	1·96843	1·98614	5	10	1·94064	2·04861	8	10	1·88815	2·20602
2	20	1·96750	1·98808	5	20	1·93840	2·05414	8	20	1·88444	2·22003
2	30	1·96649	1·99017	5	30	1·93608	2·05996	8	30	1·88064	2·23493
2	40	1·96541	1·99243	5	40	1·93368	2·06605	8	40	1·87676	2·25081
2	50	1·96426	1·99484	5	50	1·93122	2·07246	8	50	1·87278	2·26775
3	0	1·96305	1·99743	6	0	1·92866	2·07918	9	0	1·86870	2·28587
3	10	1·96176	2·00019	6	10	1·92604	2·08624	9	10	1·86454	2·30531
3	20	1·96040	2·00312	6	20	1·92333	2·09365	9	20	1·86029	2·32623
3	30	1·95897	2·00623	6	30	1·92054	2·10143	9	30	1·85593	2·34882
3	40	1·95747	2·00954	6	40	1·91767	2·10961	9	40	1·85148	2·37334
3	50	1·95589	2·01303	6	50	1·91473	2·11821	9	50	1·84692	2·40003
4	0	1·95424	2·01671	7	0	1·91170	2·12725	10	0	1·84427	2·42928
4	10	1·95252	2·02060	7	10	1·90859	2·13678	10	10	1·83752	2·46152
4	20	1·95073	2·02470	7	20	1·90539	2·14680	10	20	1·83267	2·49733

EXAMPLE.

Aug. 7. 1851, in latitude 50° 48′ N., and long. 1° 6′ W., the sun had equal altitudes at the following times by chronometer.

A.M.	P.M.
$9^h\ 25^m\ 42^s{\cdot}5$	$2^h\ 59^m\ 55^s{\cdot}6$

Required the error of chronometer on mean time at the place, and also at Greenwich.

August 7	0^h	0^m	apparent time
Equation of time .		5 +	
	0	5	mean time
Long. in time . .		4	
Greenwich, Aug. 7 .	0	9	1st date
½ Elapsed time . .	2	47	
Greenwich, Aug. 6 ..	21	22	2nd date.

	Equation of time.	Diff. for 1 hour.	
August 7 . .	$5^m\ 33^s{\cdot}03$	10^m is $\frac{1}{6}$	$0^s{\cdot}3$ sub.
	·05.		0·05
	5 32·98 +		

P.M.	14^h	59^m	$55^s{\cdot}6$
A.M.	9	25	42·5
Elapsed time . .	5	34	13·1
½ Elapsed time .	2	47	6·55

Sun's declination.

6th	16°	49′	12″	N.
7th	16	32	38	N.
		16	34	
·05048				
1·03604				
1·08652		14	45	
Declination . .	16	34	27	

	(1).		(2).
A	1·93608	B	2·05996
Cot. lat. . .	9·91147	Cot. decl. .	0·52631
Prop. log. .	1·03604	Prop. log. .	1·03604
Prop. log. .	2·88359	Prop. log. .	3·62231
			2·55 —

	14^s·2 +
	2 ·55 —
Equation of equal altitudes . .	11 ·65 +

A.M.	9^h	25^m	42^s·5
½ Elapsed time	2	47	6 ·55
	0	12	49 ·05
Equation of equal altitude .			11 ·65 +
Time by chro. at app. noon .		13	0 ·70

Apparent time at apparent noon . .	0^h	0^m	0^s
Equation of time		5	32 ·98 +
Mean time at apparent noon . . .	0	5	32 ·98
Time by chro. at apparent noon . .	0	13	0 ·70
Error of chronometer at place . .		7	27 ·72 fast

To find error on Greenwich mean time.

Mean time at apparent noon . .	0^h	5^m	32^s·98
Long. in time		4	24 ·00 +
Mean time at Greenwich	0	9	56 ·98
Time by chronometer	0	13	0 ·70
Error of chro. on Gr. mean time .		3	3 ·72 fast.

(173.) Aug. 7, 1851, in latitude 50° 48′ N., and longitude 1° 6′ W., the sun had equal altitudes at the following times by chronometer.

A.M.	P.M.
9^h 3^m 42^s·31	3^h 21^m 54^s·22

Required the error of the chronometer on Greenwich mean time.

For Elements from Nautical Almanac, see preceding example. Ans., 3^m 3^s·48 fast.

(174.) Aug. 21, 1851, in latitude 50° 48′ N., and longitude 1° 6′ W. the sun had equal altitudes at the following times by chronometer.

A.M.	P.M.
$10^h\ 49^m\ 15^s{\cdot}4$	$1^h\ 27^m\ 27^s{\cdot}6$

Required the error of the chronometer on Greenwich mean time. Ans., $1^m\ 8^s{\cdot}83$ fast.

Elements from Nautical Almanac.

Equation of time $3^m\ 1^s{\cdot}94$ + difference for $1^h\ 0^s{\cdot}606$ —
Declination 20th 12° 34′ 59″ N. 21st 12° 15′ 9″ N.

(175.) Sept. 10, 1851, in latitude 50° 48′ N., and longitude 1° 6′ W., the sun had equal altitudes at the following times by chronometer.

A.M.	P.M.
$9^h\ 45^m\ 55^s{\cdot}2$	$2^h\ 20^m\ 39^s{\cdot}9$

Required the error of the chronometer on Greenwich mean time. Ans., $3^m\ 47^s{\cdot}28$ slow.

Elements from Nautical Almanac.

Equation of time $2^m\ 58^s{\cdot}43$ + difference in $1^h\ 0^s{\cdot}866$ +
Declination 9th 5° 27′ 27″ N. 10th 5° 4′ 45″ N.

(176.) May 14, 1844, in latitude 50° 48′ N., and longitude 15° 0′ W., the sun had equal altitudes at the following times by chronometer.

A.M.	P.M.
$10^h\ 46^m\ 57^s{\cdot}0$	$1^h\ 39^m\ 42^s{\cdot}0$

Required the error of the chronometer on the mean time at the place and also on Greenwich mean time.

Ans., Fast on mean time at place $17^m\ 4^s{\cdot}7$.
Slow on Greenwich m. time $42^m\ 55^s{\cdot}3$.

Elements from Nautical Almanac.

Equation of time $3^m\ 53^s{\cdot}8$ — difference in $1^h\ 0^s{\cdot}01$ —
Declination 13th 18° 28′ 49″ N. 14th 18° 43′ 21″ N.

To find the approximate time by chronometer when the P.M. *altitudes should be observed.*

After taking the observations in the morning it will often be convenient to estimate nearly at what *time by the chronometer* the observer should prepare to take the P.M. sights. To do this the error of the chronometer on mean time *at the place* must be supposed to be known within a few minutes. Thus suppose (as in the last example) a chronometer is known to be about 17 minutes *fast* of mean time at the place, the time of the A.M. observation was by chronometer at $10^h\ 46^m\ 57^s$, equation of time 4 minutes subtractive from apparent time. It is required to find the time the chronometer will show in the afternoon when the sun has the same altitude.

Let a = estimated error of chronometer on mean time at place (supposed fast).

t = time shown by chronometer at A.M. observation.

Then $t - a$ = mean time at A.M. observation nearly.

Let E = equation of time (supposed subtractive from apparent time).

$\therefore\ t - a + E$ = apparent time at A.M. observation

$\therefore\ 12 - (t - a + E)$ = apparent time from noon.

= apparent time of P.M. observation.

$\therefore\ 12 - (t - a + E) - E$ = mean time of P.M. observation.

And $12 - (t - a + E) - E + a$ = mean time of P.M. observation *by chronometer*.

$\therefore$ Mean time of P.M. observation as shown by the chronometer $= 12 - (t - a + E) - E + a$

$= 12 - t + 2\,(a - E)$.

Thus (see ex.) let $t = 10^h\ 46^m\ 57^s$, $a = 17^m$, $E = 4^m$

$\therefore$ Time by chronometer $= 1^h\ 13^m\ 3^s + 26^m = 1^h\ 39^m$.

It appears from this that the observer need not prepare to take his P.M. sights until $1^h\ 30^m$ by chronometer.

A similar formula may be made to suit any other case.

CHAPTER VIII.

RULES FOR FINDING THE LONGITUDE BY CHRONOMETER AND BY LUNAR OBSERVATIONS.

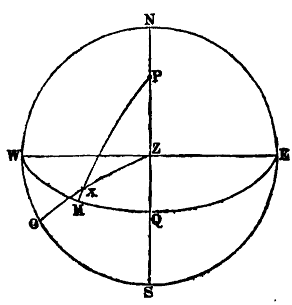
THE two principal methods for finding the longitude at sea, by astronomical observations, are by means of a chronometer, whose error is known on Greenwich mean time; or by observing the distance of the moon from some well known star, and calculating from thence Greenwich mean time: ship mean time is to be obtained in both methods by the same kind of observation. To find the longitude by chronometer, an altitude of a heavenly body is to be taken—an operation requiring very little skill in the observer. To find the longitude by lunar observations, the distance of the moon from some other heavenly body must be observed with considerable accuracy; the skill necessary to do this can only be acquired by practice: for these reasons the method of finding the longitude by chronometer is the one chiefly in use, although the longitude deduced from it depends on the regular going of a time-keeper, whose rate from various causes is continually liable to change, while the other, which in fact is (within certain limits) correct and independent of all errors of chronometer, is rarely applied. Another objection usually urged against the use of the method of finding the longitude by lunar observation, is the labour required in reducing the observations; but we will endeavour to show that this ought not to deter the student; for that the work, although certainly more laborious than that required by the other method, is simple, and no ambiguity or distinction of cases need occur to distract the observer.

From our own impression of the utility of lunars we feel it right to devote more than usual space to this method of finding the longitude, and we shall therefore give a variety of distinct rules to suit such cases as most commonly occur.

Longitude by chronometer.

When a chronometer is taken to sea, the error on Greenwich mean time, and its daily rate are supposed to accompany it: knowing then the error and rate, it is easy to determine the Greenwich mean time at any instant afterwards by applying its original error and the accumulated rate in the interval: the corresponding mean time at the ship may be found by observing the altitude of the sun, or any other heavenly body, when it bears as nearly east or west as possible. The difference between the two times is the longitude of ship.

To find the longitude by an observed altitude of the sun.



Let NWSE represent the horizon, NZS the celestial meridian, Z the zenith of the spectator, P the pole, and WQE the celestial equator. Then ZQ is the latitude, and if X be the place of the sun at the time of the observation XO is its altitude, and ZX the zenith distance; draw the circle PXM, then XM is the sun's declination known from the Nautical Almanac: hence in the triangle ZPX the three sides are known, namely, PX the polar distance, ZX the zenith distance, and PZ the colatitude, to find the hour angle ZPX, from which mean time at the ship is easily found as pointed out in p. 172.

Rule XLVII.

First. When the object observed is the sun.

1. Get a Greenwich date.

2. Find Greenwich mean time at the instant of the observation, by bringing up the error of the chronometer by Rule XLII. p. 167.

3. Take out of the Nautical Almanac both the declination of the sun and the equation of time, for the noon before and the noon after the Greenwich date; take out also the sun's semidiameter.

4. Correct the declination and equation of time for the Greenwich date (or rather for the Greenwich mean time as shown by the chronometer), either by proportional logarithms or otherwise.

5. Correct the observed altitude for index correction, dip, semi., correction in alt., and thus get the true altitude, which subtract from 90° to obtain the zenith distance.

6. *To find apparent time at ship* (using log. haversines).* Under the latitude put the sun's declination, and, if the names be alike, take the difference; but if unlike take the sum. Under the result put the zenith distance, and find the sum and difference. Add together the log. secants of the two first terms in this form (omitting the tens in each index) and the halves of the log. haversines of the two last, and (rejecting the ten in the index) look out the sum as a log. haversine, *to be taken out at the top of the page if the sun is west of the meridian, but at the bottom of the page if the sun is east of the meridian;* the result is apparent time at the ship at the instant of observation.

7. *To find ship mean time.* To the apparent time just

* If the student have no table of haversines he may proceed as pointed out in the note, p. 171, and example, p. 174, to find ship apparent time.

obtained, apply the equation of time, with its proper sign as directed in the Nautical Almanac: the result is mean time at the ship or place of observation.

8. *To find the longitude.* Under ship mean time put Greenwich mean time as known by the chronometer; the difference is the longitude in time, *west*, if the Greenwich time is greater than ship time, otherwise *east*.

EXAMPLES.

1. Sept. 23, 1845, at $4^h\ 45^m$ P.M. mean time nearly, in latitude 50° 30′ N., and longitude by account 110° 0′ W., when a chronometer showed $11^h\ 59^m\ 30^s$, the observed altitude of the sun's lower limb was 11° 0′ 50″, the index correction — 3′ 20″ and the height of the eye above the sea 20 feet; required the longitude.

On August 21 the chronometer was fast on Greenwich mean time $0^m\ 45^s{\cdot}5$ and its daily rate was $5^s{\cdot}7$ losing.

Ship, Sept. 23 . . .	4^h	45^m	Daily rate .		$5^s{\cdot}7$ losing
Long. in time . . .	7	20 W.	Interval .		$33\frac{1}{2}$
Greenwich, Sept. 23 .	12	5			171
Interval from Aug. 21 to					171
Greenwich date, $33^d\ 12^h$,			12^h is $\frac{1}{2}$ . .		188·1
or $33\frac{1}{2}$ days.					2·8
					6·0) 190·9
					$3^m\ 10^s{\cdot}9$ lost

Chronometer showed . .	11	59	30·0	
	12	2	40·9	
Original error		0	45·5	fast
Greenwich, Sept. 23 . .	12	1	55·4	

Or the mean time at Greenwich when the observation was taken.

Sun's declination.				Equation of time.			Sun's semi.
23rd . .	0°	6′	56″ S.	23rd . .	7^m	42^s·0 sub.	15′ 58″
24th . .	0	30	21 S.	24th . .	8	2·6	
		23	25			20·6	
·29983				·29983			
·88575				2·72167			
1·18558		11	45	3·02150		10·3	
Sun's decl.	0	18	41 S.		7	52·3	

Observed altitude . .	11°	0′	50″	
Index correction . . .		3	20	—
	10	57	30	
Dip		4	24	—
	10	53	6	
Semidiameter		15	58	
	11	9	4	
Correction in altitude .		4	38	—
True altitude	11	4	26	
	90			
Zenith distance . . .	78	55	34	

Latitude . . .	50°	30′	0″ N.	. .	Sec. . . .	0·197718
Declination . .	0	18	41 S.	. .	Sec. . . .	0·000006
	50	48	41			
Zenith distance .	78	55	34			
Sum	129	44	15	. . . .	½ havers. .	4·956810
Difference . .	28	6	53	. . . .	½ havers. .	4·385444
					Havers. . .	9·539978

Apparent time	4^h	48^m	35^s	
Equation of time . . .		7	52·3	—
Ship mean time	4	40	42·7	
Greenwich mean time . .	12	1	55·4	
Long. in time	7	21	12·7 W.	

∴ Long. = 110° 18′ 15″ W.

2. April 18, 1844, at 9^h 18^m A.M. mean time nearly, in latitude 50° 48′ N., and longitude by account 1° 0′ W., when a chronometer showed 9^h 27^m 48^s, the observed altitude of

the sun's lower limb was 76° 16′ 46″ (in artificial horizon), index correction 3′ 46″—; required the longitude. On April 1, the chronometer was fast 1^m $58^s \cdot 7$ on Greenwich mean time, and its mean daily rate was $11^s \cdot 2$ gaining.

Ship, April 17	21^h	18^m
Long. in time		4
Greenwich, April 17 . .	21	22

Interval from April 1 to Greenwich date, 16^d $21\frac{1}{2}^h$.

Sun's declination.					Equation of time.			
17th . . .	10°	36′	49″	N.	17th . . .	0^m	$31^s \cdot 3$	sub.
18th . . .	10	57	46	N.	18th . . .	0	45·1	
		20	57				13·8	
·05020					·05020			
·93409					2·89354			
·98429		18	40		2·94374		12·3	
	10	55	29	N.		0	43·6	sub.

Sun's semi. 15′ 56″.

Daily rate . . .			$11^s \cdot 2$	
			16	
			672	
			112	
12^h is $\frac{1}{2}$ . . .			179·2	
6 „ $\frac{1}{4}$ . . .			5·6	
$1\frac{1}{2}$ „ $\frac{1}{4}$ . . .			2·8	
			·7	
			6,0) 188·3	
		3^m	$8^s \cdot 3$	gained
Chronometer showed	9	27	48·0	
	9	24	39·7	
Original error		1	58·7	fast
Greenwich mean time. . .	9	22	41·0	
Add	12			(p. 71.)
Greenwich mean time . .	21	22	41·0	

Observed altitude . . .		76°	16′	46″
			3	46 —
	2)	76	13	0
		38	6	30
Semidiameter			15	56
		38	22	26
Correction in altitude .			1	7 —
True altitude		38	21	19
		90		
Zenith distance . . .		51	38	41

Latitude . . .	50°	48′	0″	. . .	Sec. . .	0·199263
Declination . .	10	55	29	. . .	Sec. . .	0·007943
	39	52	31			
Zenith distance .	51	38	41			
Sum	91	31	12	. . .	½ hav. . .	4·855173
Difference . .	11	46	10	. . .	½ hav. . .	4·010890
					Hav. . .	9·073269

Apparent time . . .	21h	19m	0s·0
Equation of time . .		0	43·6 —
Ship mean time . .	21	18	16·4
Greenwich mean time	21	22	41·0
Long. in time . . .		4	24·6

Longitude . . 1° 6′ 9″ W.

(177.) Sept. 25, 1845, at 4^h 20^m P.M., mean time nearly, in latitude 59° 30′ N., and longitude by account 112° 30′ W., when a chronometer showed 11^h 44^m 20^s, the observed altitude of the sun's lower limb was 10° 50′ 10″, the index correction + 6′ 10″ and height of eye above the sea 18 feet, required the longitude. On Sept. 20, the chronometer was fast on Greenwich mean time 0^m $30^s{\cdot}7$ and its daily rate was $10^s{\cdot}5$ losing. Ans., 112° 33′ W.

(178.) May 30, 1845, at 3^h 10^m P.M., mean time nearly, in latitude 30° 12′ 0″ S., and longitude by account 156° 0′ E., the observed altitude of the sun's lower limb was 21° 8′ 40″ when a chronometer showed 4^h 44^m 56^s; the

index correction — 1′ 10″, and height of eye above the sea 30 feet; required the longitude. On May 19, the chronometer was fast 5^m 16^s on Greenwich mean time, and its daily rate was $3^s{\cdot}5$ gaining. Ans., 156° 20′ E.

(179.) July 8, 1849, at 1^h 40^m P.M., mean time nearly, in latitude 50° 48′ N., and longitude by account 1° 1′ W., the observed altitude of the sun's lower limb, taken by the artificial horizon, was 109° 54′ 44″ the chronometer showed 1^h 44^m 14^s, the index correction + 1′ 25″; required the longitude. On July 1, the chronometer was slow on Greenwich mean time 8^m $18^s{\cdot}4$, and its daily rate was $3^s{\cdot}5$ losing. Ans., 1° 6′ 0″ W.

(180.) January 20, 1846 at 6^h 40^m A.M., mean time nearly, in latitude 56° 20′ S., and longitude by account 83° 10′ W., when a chronometer showed 0^h 14^m 50^s, the observed altitude of the sun's lower limb was 20° 20′ 30″, the index correction — 1′ 30″, and the height of the eye above the sea 20 feet; required the longitude. On Jan. 2, the chronometer was fast on Greenwich mean time 5^m 20^s, and on Jan. 6 it was fast 4^m 52^s, from which may be found its mean daily rate. Ans., 83° 5′ 45″ W.

(181.) Feb. 10, 1846, at 7^h 50^m, A.M., mean time nearly, in latitude 50° 48′ N., and longitude by account 170° 30′ E., when a chronometer showed 9^h 59^m 25^s, the observed altitude of the sun's lower limb was 51° 9′ 10″, the index correction — 3′ 20″, and the height of eye above the sea 16 feet, required the longitude. On Jan. 31, at Greenwich noon, the chronometer was fast 1^h 34^m 43^s, and its daily rate was $20^s{\cdot}6$ losing. Ans., 170° 34′ 15″ E.

Elements from Nautical Almanac.

	Sun's declination.	Equation of time.	Semi.
Sept. 25	0° 53′ 47″ S.	8^m $23^s{\cdot}0$ sub.	15′ 59″
„ 26	1 17 12 S.	8 43·4	
May 29	21 38 43 N.	2 56·4 sub.	15 47
„ 30	21 47 47 N.	2 48·5	

	Sun's declination.	Equation of time.	Semi.
July 8 . . .	22° 29′ 9″ N. .	4ᵐ 40ˢ·9 add . .	15 45
„ 9 . . .	22 22 7 N. .	4 50·1	
Jan. 20 . . .	20 8 22 S. .	11 19·2 add . .	16 16
Feb. 9 . . .	14 41 33 S. .	14 31·0 add . .	16 13
„ 10 . . .	14 22 11 S. .	14 32·0	

Rule XLVIII.

To find the longitude by star chronometer.

Object observed, a star.

1. Get a Greenwich date.

2. Find Greenwich mean time by bringing up the error of the chronometer to the instant of observation by Rule XLII.

3. Take out of the Nautical Almanac the right ascension and declination of the star, and also the right ascension of the mean sun (called in Nautical Almanac sidereal time), for mean noon of the Greenwich date.

4. Correct the right ascension of the mean sun for Greenwich date.

5. Correct the observed altitude for index correction, dip, and refraction, and thus get the true altitude, which subtract from 90° to obtain the zenith distance.

6. *To find the star's hour angle* (using log. haversines*). Under the latitude put the star's declination; add if the names be unlike, subtract if like; under the result put star's zenith distance, and take the sum and difference. Add together the log. secants of the two first terms in this form (omitting the tens in each index), and the halves of the log. haversines of the two last, the sum rejecting ten in the index, will be the log. haversines of star's

* If the student have no table of haversines, he may proceed as directed in the note to p. 171, and Ex. p. 174, to find the sun's hour angle or apparent time, using the star's declination instead of sun's, so as to get the star's hour angle.

hour angle, to be taken out at top of page, if heavenly body be west of meridian, but at bottom, if east of meridian.

7. *To find mean time at ship.* To the hour angle thus found, add the star's right ascension; and from the sum, increased if necessary by 24 hours, subtract the right ascension of the mean sun; the remainder is mean time at the place at the instant of observation.

8. *To find the longitude.* Under ship mean time put Greenwich mean time as known by the chronometer; the difference is the longitude in time, *west*, if Greenwich time is greater than ship time, otherwise *east*.

EXAMPLES.

1. Sept. 10, 1844, at 7^h 15^m P.M., mean time nearly, in latitude 48° 20′ N., and longitude by account 32° E., when a timekeeper showed 5^h 1^m 28^s, the observed altitude of α Bootis (Arcturus) W. of meridian was 31° 5′ 40″, the index correction — 4′ 10″, and height of eye above the sea 20 feet; required the longitude. On Aug. 25, the chronometer was slow on Greenwich mean time 2^m 40^s, and its daily rate was 4^s·3 gaining.

Ship, Sept. 10	7^h	15^m	
Long. in time	2	8	E.
Greenwich, Sept. 10 . .	5	7	

Interval, 16^d 5^h.

Daily rate . . .	4^s·3
	16
	258
	43
4^h is $\frac{1}{6}$ . .	68·8
1 „ $\frac{1}{4}$ . .	0·7
	0·2
	6,0) 69·7

Accumulated rate		1^m.	9^s·7 gained
Chronometer showed . . .	5	1	28 ·0
	5	0	18 ·3
Original error		2	40 ·0 slow
Greenwich mean time . . .	5	2	58 ·3

Observed altitude . . .	31°	5′	40″	
Index correction		4	10	—
	31	1	30	
Dip		4	24	—
	30	57	6	
Refraction		1	37	—
	30	55	29	
	90			
Zenith distance	59	4	31	

Right ascension mean sun.

10th	11^h	18^m	$28^s{\cdot}15$
5^h . .			49·28
3^m . .			·49
	11	19	17·9

Star's right ascension . . . . 14^h 8^m $34^s{\cdot}65$

Star's declination . . . . . 19° 59′ 44″ N.

Latitude . . .	48°	20′	0″	N. . .	Sec. . .	0·177312
Declination . .	19	59	44	N. . .	Sec. . .	0·027003
	28	20	16			
Zenith distance .	59	4	31			
Sum	87	24	47	. . .	½ hav. . .	4·839453
Difference . . .	30	44	15	. . .	½ hav. . .	4·423295
					Hav. . .	9·467063

Hour angle	4^h	22^m	15^s
Star's right ascension . .	14	8	34·6
	18	30	49·6
Right ascen. mean sun . .	11	19	17·9
Ship mean time	7	11	31·7
Greenwich mean time . .	5	2	58·3
Longitude in time . . .	2	8	33·4

Longitude . . 32° 8′ 21″ E.

2. May 24, 1844, at 11^h 11^m P.M., mean time nearly, in latitude 50° 48′ N., and longitude by account 1° 0′ W. when a timekeeper showed 11^h 12^m $11^s{\cdot}8$, the observed altitude of *α* Lyræ (Vega) E. of meridian was 109° 29′ 18″ in

artificial horizon, the index correction — 3′ 46″, required the longitude. On May 14, at Greenwich mean noon the chronometer was slow $1^m\ 15^s{\cdot}8$ and its mean daily rate was $7^s{\cdot}4$ losing.

Ship, May 24	11^h	11^m
Long. in time		4
Greenwich, May 24 . .	11	15

Daily rate . . .		$7^s{\cdot}4$
Interval, $10^d\ 11^h\ 15^m$		10
	8^h is $\frac{1}{3}$	74·0
	2 „ $\frac{1}{4}$	2·5
	1 „ $\frac{1}{2}$	·6
	15^m „ $\frac{1}{4}$	·1
	6,0)	77·2 lost

Accumulated rate . . .		1^m	$17^s{\cdot}2$
Chronometer showed . .	11	12	11 ·8
	11	13	29 ·0
Original error		1	15 ·8
Greenwich mean time . .	11	14	44 ·8

Observed altitude . .	109°	29′	18″	
Index correction . . .		3	46	—
	2) 109	25	32	
	54	42	46	
Refraction			41	—
	54	42	5	
	90			
Zenith distance . . .	35	17	55	

Right ascension mean sun.

24th	4^h	8^m	$43^s{\cdot}56$
11^h . .		1	48 ·42
14^m . .			2 ·30
45^s . .			13
	4	10	34 ·4

Star's right ascension . .	18^h	31^m	$42^s{\cdot}2$	
Star's declination. . . .	38°	38′	24″	N.

Latitude . . .	50° 48′ 0″ N. . .	Sec. . .	0·199263			
Declination . .	38 38 24 N. . .	Sec. . .	0·107300			
	12 9 36					
	35 17 55					
Sum.	47 27 31 . . .	½ hav. . .	4·604673			
Difference. . .	23 8 19 . . .	½ hav. . .	4·302209			
			9·213445			
	Hour angle			20^h	49^m	13^s
	Star's right ascension . .			18	31	42·2
				39	20	55·2
	Right ascen. mean sun . .			4	10	34·4
	Ship mean time			11	10	20·8
	Greenwich mean time . .			11	14	44·8
	Longitude in time . . .				4	24·0

Longitude = 1° 6′ W.

(182.) Aug. 20, 1845 at 0^h 30^m A.M., mean time nearly, in latitude 50° 20′ N. and longitude by account 142° 0′ E. when a chronometer showed 2^h 41^m 12^s, the observed altitude of the star α Aquilæ (Altair) was 36° 59′ 50″, west of the meridian, the index correction + 6′ 30″ and height of eye above the sea 20 feet; required the longitude. On Aug. 1 the chronometer was slow on Greenwich mean time, 17^m 45^s·0 and its daily rate was 4^s·3 losing.

Ans., 142° 14′ 15″ E.

(183.) Sept. 10, 1844, at 4^h 21^m A.M., mean time nearly, in latitude 40° 36′ N., and longitude by account 73° E., when a chronometer showed 11^h 21^m 56^s the observed altitude of β Geminorum (Pollux) was 39° 0′ 10″ east of meridian, the index correction — 4′ 10″ and height of eye above the sea 20 feet, required the longitude. On Aug. 20, the chronometer was slow on Greenwich mean time 3^m 19^s·9, and its daily rate was 9^s·3 gaining.

Ans., 72° 45′ 45″ E.

(184.) January 16, 1845, at 8^h 0^m P.M., mean time nearly, in latitude 49° 56′ 50″ N., and longitude by account 94° 30′ W., when a chronometer showed 2^h 24^m 30^s, the

observed altitude of α Leonis (Regulus) was $8^h\ 4'\ 20''$, E of meridian, the index correction — $4'\ 20''$, and height of eye above the sea 25 feet, required the longitude. On January 1, the chronometer was fast on Greenwich mean time $5^m\ 30^s{\cdot}5$ and its daily rate was $5^s{\cdot}5$ losing.

Ans., $94°\ 24'\ 45''$ W.

(185.) January 20, 1846, at $8^h\ 30^m$ P.M., mean time nearly, in latitude $50°\ 48'$ N., and longitude by account $7°\ 10'$ W., when a chronometer showed $8^h\ 32^m\ 50^s$ the observed altitude of ϵ Leonis was $28°\ 0'\ 10''$ east of the meridian, the index correction — $6'\ 20''$, and the height of eye above the sea 20 feet; required the longitude. On January 2, the chronometer was fast on Greenwich mean time $30^m\ 30^s$ and its mean daily rate was $15^s{\cdot}5$ losing. Ans., $7°\ 18'$ E.

Elements from Nautical Almanac.

	Right ascen. mean sun.	Star's right ascen.	Star's decl.
Aug. 19	$9^h\ 50^m\ 46^s{\cdot}5$	$19^h\ 43^m\ 17^s{\cdot}0$	$8°\ 28'\ 7''$ N.
Sept. 9	11 14 31·6	7 35 48·6	28 23 40 N.
Jan. 16	19 43 7·2	10 0 8·8	12 43 6 N.
Jan. 20	19 57 55·9	9 37 8·3	24 28 33 N.

Longitude by lunar observation.

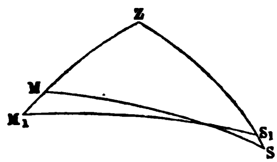
The *time at the ship* is obtained by the same kind of observation as that for finding the longitude by chronometer. The *time at Greenwich* is found by calculating the true distance of the moon from the sun or some other heavenly body, and comparing it with the distance of the moon from the same heavenly body as recorded in the Nautical Almanac for some given time at Greenwich.

To find the true distance.

The true distance is found by clearing the observed distance of the effects of parallax and refraction, by the following or some other similar methods.

Rule for clearing the distance by the common logarithmic tables.

Let $S_{,}$ be the apparent place of the sun, then in consequence of the effects of parallax and refraction, the latter raising the sun more than the former, depressing it in altitude, its true place will be below $S_{,}$ as at S.



Let $M_{,}$ be the apparent place of the moon, then its *true* place will be above $M_{,}$ since the moon is depressed by parallax considerably more than it is raised by refraction; let therefore M represent the true place of the moon.

Through $M_{,}$ and $S_{,}$ draw the arc of a great circle $M_{,}S_{,}$; this will be the apparent distance found *by observation*, draw also M S an arc of a great circle through the true places of the sun and moon, then the arc M S will be the true distance of the heavenly bodies at the time of observation *to be computed.*

Let Z be the zenith of the spectator; then if we suppose the effect of parallax to take place in a vertical circle, the arcs Z $M_{,}$ Z S are circles of altitude.

The true distance M S may be computed by means of the common rules of spherical trigonometry, as follows:

1. In triangle $M_{,}$ Z $S_{,}$ the three sides are given, namely, the two apparent distances Z $M_{,}$ and Z $S_{,}$ and the observed distance $M_{,}$ $S_{,}$, to find the angle Z.

2. In triangle M Z S are given the two sides M Z and S Z the true zenith distances of the heavenly bodies and the included angle Z just found, to compute the third side M S, *the true distance required.*

The practical inconvenience of this method arises from the necessity of taking out the log. sines, &c., to the nearest second, a work of considerable labour with the mmon tables of logarithm sines, &c., which seldom give

the arcs nearer than 15″. To obviate this the true distance is now usually found in terms of the versines, the arcs in the table of versines being given to the nearest second.

Investigation of the Rule for clearing the distance by means of a table of versines.

Let Z M the true zenith distance of moon $= z$
„ Z S „ of sun or star $= z_{,}$
„ Z M, the app. zenith distance of moon $= 90 - a$
„ Z S, „ of sun or star $= 90 - a_{,}$
(where a and $a_{,}$ are the apparent altitudes of moon and sun or star)
M, S, apparent distance of bodies . . . $= d$
and M S their true distance . . . . . $= x$

$$\text{In triangle Z M S} \quad \text{cos. } z = \frac{\text{cos. } x - \text{cos. } z \text{ cos. } x}{\text{sin. } z \text{ sin. } z_{,}}$$

$$\text{In triangle Z M, S,} \quad . \quad \text{cos. } z = \frac{\text{cos. } d - \text{sin. } a \text{ sin. } a_{,}}{\text{cos. } a \text{ cos. } a_{,}}$$

$$\therefore \frac{\text{cos. } x - \text{cos. } z \text{ cos } z_{,}}{\text{sin. } z \text{ sin. } z_{,}} = \frac{\text{cos. } d - \text{sin. } a \text{ sin. } a_{,}}{\text{cos. } a \text{ cos. } a_{,}}$$

adding 1 to both sides, and multiplying up

$$\frac{\text{cos. } x - (\text{cos. } z \text{ cos. } z_{,} - \text{sin. } z \text{ sin. } z_{,})}{\text{sin. } z \text{ sin. } z_{,}}$$

$$= \frac{\text{cos. } d - \text{sin. } a \text{ sin. } a_{,} + \text{cos. } a \text{ cos. } a_{,}}{\text{cos. } a \text{ cos. } a_{,}}$$

$$\text{or } \frac{\text{cos. } a - \text{cos. } (z + z_{,})}{\text{sin. } z \text{ sin. } z_{,}} = \frac{\text{cos. } d + \text{cos. } (a + a_{,})}{\text{cos. } a \text{ cos. } a_{,}}$$

$$\therefore \text{cos. } x - \text{cos. } (z + z_{,})$$

$$= (\text{cos. } d + \text{cos. } \overline{a + a}). \frac{\text{sin. } z \text{ sin. } z_{,}}{\text{cos. } a \text{ cos. } a_{,}}$$

$$= (\text{cos. } a + \text{cos. } \overline{a + a_{,}}). \ 2 \text{ cos. A}$$

$$\left(\text{Assuming } \frac{\text{sin. } z \text{ sin. } z_{,}}{\text{cos. } a \text{ cos. } a_{,}} = 2 \text{ cos. A}\right) \therefore \text{cos. } x - \text{cos. } (z + z_{,})$$

$$= 2 \text{ cos. } d \text{ cos. A} + 2 \text{ cos. } (a + a_{,}) \text{ cos. A}$$

$= \cos.(a + A) + \cos.(d - A) + \cos.(a + a_{,} + A) + \cos.(a + a_{,} - A)$: transposing cos. $(z + z_{,})$ and subtracting each term from 1 we have

$$1 - \cos. x = 1 - \cos. (z + z_{,}) + 1 - \cos. (a + A) + 1 - \cos. (d - A) + 1 - \cos. (a + a_{,} + A) + 1 - \cos. (a + a_{,} - A) - 4.$$

Or in tabular versines (see the author's Trigonometry, p. 30.)

$$\therefore \text{tab. ver. } x = \text{tab. ver. } (z + z_{,}) + \text{tab. ver. } (d + A) + \text{tab. ver. } (d - A) + \text{tab. ver. } (a + a_{,} + A) + \text{tab. ver. } (a + a_{,} - A) - 4000000.$$

The auxiliary angle A is found in the Nautical Tables of Inman, Riddle, Norie, and others.

The student will be able to determine the relative value of the two methods, by working an example by each.

EXAMPLE.

Required the true distance of the moon from the sun, having given

App. alt. sun . .	34° 21′ 32″	True alt. sun . .	34° 20′ 14″
App. alt. moon . .	57 11 25	True alt. moon .	57 40 11
And apparent distance of centres			35 47 24

First method.—By the common rules of trigonometry.

1. *To find angle* $z_{,}$ *in triangle* $s_{,}zm_{,}$.

Sun's app. zenith dist. .	55°	38′	28″	. Cosec. . . .	0·0832731
Moon's app. zenith dist. .	32	48	35	. Cosec. . . .	0·2661203
	22	49	53		
Apparent distance . . .	35	47	24		
Sum	58	37	17		
Difference	12	57	31		
½ sum	29	18	38·5	. Sin	9·6897928
½ difference	6	28	45·5	. Sin	9·0524798
					2) 19·0916660
				Sin ½ z . . .	9·5458330
				½ z . . .	20 34 28·5
					2
				z =	41 8 57

2. *To find* S M *in triangle* S Z M.

Const. log.	6·3010300
Sin. sun's true zenith dist. . .	9·9168391
Sin. moon's true zenith dist. .	9·7281939
Twice sin. ½ z	19·0916660
Log. ver. arc . . .	5·0377290
Vers. arc.	109076
Ver. diff. of zenith distances .	81669
	107
Vers. distance	190852

∴ True distance = 35° 59′ 14″.

Second method.—The true distance found by versines, the auxiliary angle (taken from the table) being 60° 25′ 16″.

							Parts for seconds.
Sun's true zen. dist. . .	55°	39′	46″				
Moon's true zen. dist. .	32	19	50				
Sum . . .	87	59	36	vers. . .	964810	. .	174
					1107999	. .	195
Apparent distance . .	35	47	24		90885	. .	104
Aux. angle (A) . . .	60	25	16		1882674	. .	30
Sum . . .	96	12	40	vers.	148888	. .	102
Difference .	24	37	52	vers.	4190251	. .	605
Sun's apparent alt. . .	57	11	25		605		
Moon's apparent alt. .	34	21	32		4190856		
	91	32	57		4000000		
Aux. angle (A) . . .	60	25	16		vers. 190856		
Sum . . .	151	58	13	vers.	∴ True dist. 35° 59′ 15″		
Difference .	31	7	41	vers.			

In practice it is not necessary to take from the table of versines more than the last five figures, rejecting also all but these last five in the sum, since the true distance will be always either in the same column with the apparent distance or the adjacent one. Thus, taking the preceding example, it may be worked thus:—

55°	39′	46″	
32	19	50	
87	59	36	vers.
35	47	24	
60	25	16	
96	12	40	vers.
24	37	52	vers.
57	11	25	
34	21	32	
91	32	57	
60	25	16	
151	58	13	vers.
31	7	41	vers.

Vers.	Parts for seconds.
64810 . . .	174
07999 . . .	195
90885 . . .	104
82674 . . .	30
43883 . . .	102
90251 . . .	605
605	
90856 . .	35° 59′
812	
44 . .	15″
True dist. . .	35 59 15

Hence this rule for clearing the distance by means of an auxiliary angle.

Rule XLIX.

To clear the lunar distance.

1. Under the sun's or star's true zenith distance put the moon's true zenith distance; take the sum which mark vers.

2. Under the apparent distance of the two centres put the auxiliary angle A; take their sum and difference, against both, which mark vers.

3. Under the sun's or star's apparent altitude put the moon's apparent altitude and take their sum; under which put the auxiliary angle A; take the sum and difference, against both which mark vers.

4. Add together the five last figures of the versines of the quantities marked vers., rejecting all but the last five in the result, which look for in the column of versines under the apparent distance, or under the adjacent one: take out the arc corresponding thereto, which will be the true distance required. See example, above.

EXAMPLES.

(186.) The apparent altitude of the moon	=	50° 54′ 38″
„ true zenith distance „	=	38 30 40
„ apparent altitude of the sun	=	30 29 48
„ true zenith distance „	=	59 31 44
and the apparent distance of the two centres	=	88 49 58
and the auxiliary angle A	=	60 24 12
required the true distance.	Ans.,	88 24 17

(187.) The sun's apparent altitude . .	=	54° 29′ 33″
„ moon's apparent altitude . .	=	5 25 59
„ moon's zenith distance . .	=	83 48 29
„ sun's zenith distance . . .	=	35 31 3
„ auxiliary angle A	=	60 2 11
„ apparent distance	=	105 5 47
required the true distance.	Ans.,	104 26 18

(188.) The sun's apparent altitude is 17° 39′ 31″, the moon's apparent altitude 24° 13′ 45″, the moon's zenith distance 64° 56′ 45″, the sun's zenith distance 72° 23′ 22″, the auxiliary angle A 60° 12′ 33″, and the apparent distance 111° 20′ 45″; required the true distance.

Ans., 110° 56′ 0″.

(189.) The sun's apparent altitude is 54° 47′ 4″, the moon's apparent altitude 21° 20′ 1″ the moon's zenith distance 67° 51′ 5″, the sun's zenith distance 35° 13′ 32″, the auxiliary angle A 60° 10′ 44″, and the apparent distance 71° 16′ 44″; required the true distance.

Ans., 70° 38′ 5″.

(190.) The sun's apparent altitude is 12° 19′ 30″, the moon's apparent altitude 20° 40′ 18″, the moon's zenith distance 68° 28′ 19″, the sun's zenith distance 77° 44′ 42″, the auxiliary angle A 60° 10′ 53″, and the apparent distance 124° 44′ 32″; required the true distance.

Ans., 124° 19′ 11″.

(191.) The sun's apparent altitude is 57° 53′ 52″, the moon's apparent altitude 35° 3′ 2″, the moon's zenith

distance 54° 11′ 56″, the sun's zenith distance 32° 6′ 40″, the auxiliary angle A 60° 17′ 54″, and the apparent distance 65° 34′ 42″; required the true distance.

Ans., 64° 58′ 10″.

(192.) The sun's apparent altitude is 15° 43′ 48″, the moon's apparent altitude 16° 5′ 5″, the moon's zenith distance 73° 1′ 32″, the sun's zenith distance 74° 19′ 28″, the auxiliary angle A 60° 8′ 36″, and the apparent distance 119° 44′ 31″; required the true distance.

Ans., 119° 19′ 51″.

(193.) The apparent altitude of a star is 20° 13′ 26″, the moon's apparent altitude 31° 17′ 22″, the star's zenith distance 69° 49′ 11″, the moon's zenith distance 57° 57′ 44″, the auxiliary angle A 60° 15′ 21″, and the apparent distance 72° 42′ 16″; required the true distance.

Ans., 72° 33′ 4″.

(194.) The apparent altitude of a star is 29° 59′ 16″, the moon's apparent altitude 32° 30′ 10″, the star's zenith distance 60° 2′ 24, the moon's zenith distance 56° 41′ 33″, the auxiliary angle A 60° 17′ 23″, and the apparent distance 58° 44′ 19″, required the true distance.

Ans., 58° 30′ 21″.

Rule L.

To find the longitude by lunar observations.

Objects observed, sun and moon. Altitudes taken. Ship mean time determined from sun's altitude.

1. Get a Greenwich date.

2. Take from the Nautical Almanac and correct for Greenwich date the following quantities:—

Sun's declination and semidiameter.

Equation of time (noting whether it is to be added to or subtracted from the ship apparent time).

Moon's semidiameter and horizontal parallax.

3. Correct the sun's apparent altitude for index correction, dip, semidiameter, correction in altitude, and thus get the sun's apparent and true altitudes. Subtract the true altitude from 90° for sun's zenith distance.

4. Correct the moon's observed altitude for index correction, dip, semidiameter (augmented), correction in altitude, and thus get the moon's apparent and true altitude. Subtract the true altitude from 90° for moon's zenith distance.

5. When the moon's correction in altitude is taken out of the Tables, take out also at the same opening the auxiliary angle A.

6. Correct the observed distance for index correction, and to the result add the semidiameter of the sun and moon (augmented), and thus get the apparent distance of the centres.

7. *To find ship mean time.** Under sun's declination put the latitude of the ship; take the *sum* if their names be *unlike*, the *difference* if the names be *alike*. Under the result put the sun's zenith distance; take the sum and difference of the last two lines put down. Add together the log. secants of the two first quantities in this form (omitting to put down the tens in the index) and half of the log. haversines of each of the two last quantities. The sum will be the log. haversine of the ship apparent time. When the sun is west of the meridian, the time corresponding to the haversine must be taken out at the top of the page; but when the sun is east it must be taken out at the bottom. The result is apparent time at the ship: to this apply the equation of time with its proper sign and the result will be the ship mean time.

8. *To calculate the true distance, and thence Greenwich mean time.*

* If the student have no table of haversines he may proceed as pointed out in the note p. 171, and Ex. p. 174, to find ship apparent time.

Add together the zenith distances of the sun and moon, and mark the sum *v*.

Add together the apparent altitudes of the sun and moon, and under the sum put the auxiliary angle A: take the sum and difference of the last two quantities, and mark each with the letter *v*.

Under the apparent distance of the centres put the auxiliary angle A and take the sum and difference and mark each result with the letter *v*.

Add together the five last figures of the versines of each of the quantities marked *v*. The five last figures in the sum being looked for in the column of versines under the apparent distance or in the adjacent column, the arc corresponding thereto will be the true distance of the sun and moon at the time of the observation.

9. *To find Greenwich mean time corresponding to this true distance.*

Take out of the Nautical Almanac two distances of the sun and moon three hours apart, between which is the true distance just calculated: place the first distance taken out under the true distance, and the one three hours after under the other distance taken out. Take the difference between the first and second, and also between the second and third. From the proportional logarithm of the first difference subtract the proportional logarithm of the second difference; the remainder is the proportional logarithm of a portion of time, which take from the table, and add thereto the hours corresponding to the first distance taken out of the Nautical Almanac. The result is Greenwich mean time when the observation was taken.

The difference between ship mean time found above and Greenwich mean time is the longitude in time; turn it into degrees, and mark it "east if the Greenwich time is the least, and west if the Greenwich time is best."

EXAMPLES.

Feb. 12, 1848, at $2^h\ 36^m$ P.M., mean time nearly, in lat. 53° 30′ N., and long. by account 15° 30′ E., the following lunar observation was taken :—

	Obs. alt. sun's L. L.	Obs. alt. moon's L. L.	Obs. dist. N. L.
	29° 17′ 16″	25° 40′ 20″	99° 27′ 30″
Index cor.	2 10	1 10	0 50 —

The height of the eye above the sea was 20 feet; required the longitude.

Ship, Feb. 12. . . .	2^h	36^m
Long. in time . . .	1	3 E.
Greenwich, Feb. 12 .	1	33

Sun's declin.		Equa. of time.
12th . .	13° 52′ 18″ S. . .	. $14^m\ 33^s$ add
13th . .	13 32 21 S. . .	. 14 32
	19 57	1
1·18985		∴ cor. 0
·95533		14 33
2·14518	1 17	
	13 51 1 S.	
	Sun's semi. .	16′ 18″

Moon's semi.			Hor. par.
12th noon .	15′ 58″		58′ 36″
„ mid. .	15 54		58 23
	4		13
·88885		·88885	
3·43136		2·91948	
4·32021	0·4	3·80833	1·7
	15 57·6		58 34·3
Aug. . .	7·4		
	16 5		

Sun's alt.		Moon's alt.	
Obs. alt. . .	29° 17′ 26″	Obs. alt. . .	25° 40′ 20″
In. cor. . .	2 10 —	In. cor. . .	1 10
	29 15 16		25 39 10
Dip	4 24 —	Dip. . . .	4 24
	29 10 52		25 34 46
Semi. . . .	16 18	Semi. . . .	16 5
	29 27 5		25 50 51
Cor. in alt. .	1 35 —	Cor. in alt. .	50 13
			31
	29 25 30		26 41 35
	90		90
Sun's Z. D. .	60 34 30	Moon's Z.D.	63 18 25

Distance.	
	99° 27′ 30″
In. cor.. . .	0 50 —
	99 26 40
Sun's semi. .	16 18
Moon's semi.	16 5
	99 58 58
Aux. angle.	
	60° 13′ 40
	9
	3
	60 13 52

To find ship mean time.

Sun's declination . .	13°	51′	1″ S. .	Sec. . .	0·012814
Latitude	53	30	0 N. . .	Sec. . .	0·225612
	67	21	1		
Sun's zenith dist. .	60	34	30		
Sum	127	55	31 . . .	½ hav. .	4·953521
Difference	6	46	31 . . .	½ hav. .	3·771503
				Hav. . .	8·963450

Apparent time . .	2^h	21^m	12^s
Equation of time . .		14	33 +
Ship mean time . .	2	35	45

To find Greenwich mean time.

Sun's zenith dist.	60°	34′	30″
Moon's zen. dist.	63	18	25
	133	52	55 vers.
Sun's app. alt. .	29	27	5
Moon's app. alt.	25	50	51
Sum	55	17	56
Aux. angle A .	60	13	52
Sum	115	31	48 vers.
Difference . .	4	55	56 vers.
App. dist. . .	99°	58′	58″
Aux. angle A. .	60	13	52
Sum	160	12	50 vers.
Difference . .	39	45	6 vers.

Versines.

57262 . .	222
30774 . .	210
03680 . .	19
40881 . .	83
31158 . .	18
63755 . .	552
552	
64307	
187	
120	

True distance.

99°	27′	25″	
98	38	0	0 hours
100	14	7	3 hours
0	49	25	
1	36	7	
·56140			
·27247			

	1^h	32^m	33^s
	0		
Greenwich mean time	1	32	33 . . . . ·28893
Ship mean time . .	2	35	45
Long. in time . . .	1	3	12

Longitude . . 15° 48′ 0″ E.

(195.) March 25, 1847, at $3^h 30^m$ P.M., mean time nearly, in lat. 52° N., and long. by account 33° W., the following lunar was taken :—

	Obs. alt. sun's L. L.	Obs. alt. moon's L. L.	Obs. dist. N. L.
	23° 10′ 20″	23° 50′ 10″	112° 56′ 30″
Index cor.	6 10 —	5 0 +	4 20 —

The height of the eye above the sea was 20 feet; required the longitude. Ans., 32° 59′ 30″ W.

(196.) April 20, 1847, at $2^h 0^m$ P.M., mean time nearly, in lat. 50° 50′ N., and long. by account 1° 40′ E., the following lunar was taken :—

	Obs. alt. sun's L. L.	Obs. alt. moon's L. L.	Obs. dist. N. L.
	43° 16′ 30″	24° 39′ 20″	69° 12′ 0″
Index cor.	0 10 +	0 20 —	0 50 —

The height of the eye above the sea was 20 feet; required the longitude. Ans., 1° 20′ 30″ E.

(197). May 19, 1847, at $2^h 50^m$ P.M., mean time nearly, in lat. 51° 30′ N., and long. by account 20° 40′ E., the following lunar was taken :—

	Obs. alt. sun's L. L.	Obs. alt. moon's L. L.	Obs. dist. N. L.
	42° 50′ 30″	25° 10′ 20″	61° 40′ 20″
Index cor.	4 10 +	6 10 —	2 10 +

The height of the eye above the sea was 20 feet; required the longitude. Ans., 20° 42′ E.

(198). Feb. 6, 1851, at $3^h 30^m$ P.M., mean time nearly, in lat. 60° 20′ N., and long. by account 26° 45′ E., the following lunar was taken :—

	Obs. alt. sun's L. L.	Obs. alt. moon's L. L.	Obs. dist. N. L.
	24° 20′ 0″	33° 10′ 0″	57° 30′ 10″
Index cor.	2 30 +	1 20 +	2 0 +

The height of the eye above the sea was 11 feet; required the longitude. Ans., 28° 35′ E.

(199). Feb. 20, 1850, at $3^h\ 50^m$ P.M., mean time nearly, in lat. 10° 20′ N., and long. by account 7° W., the following lunar was taken:—

	Obs. alt. sun's L. L.	Obs. alt. moon's L. L.	Obs. dist. N. L.
	30° 15′ 40″	24° 10′ 10″	100° 55′ 10″
Index cor.	3 10 —	1 10 +	0 30 +

The height of the eye above the sea was 20 feet; required the longitude. Ans., 7° 4′ W.

(200.) Jan. 9, 1851, at $2^h\ 50^m$ P.M., mean time nearly, in lat. 56° 10′ 20″ N., and long. by account 20° 40′ E., the following lunar was taken:—

	Obs. alt. sun's L. L.	Obs. alt. moon's L. L.	Obs. dist. N. L.
	19° 10′ 20″	25° 30′ 10″	77° 10′ 20″
Index cor.	2 10 +	1 20 —	2 20 —

The height of the eye above the sea was 20 feet; required the longitude. Ans., 20° 35′ E.

Elements from Nautical Almanac.

	Sun's declin.	Eq. of time.	Moon's semi.	Hor. par.	Sun's semi-
Mar. 25	1° 40′ 56″ N.	$6^m\ 18^s$·8 add	14′ 59″	54′ 59″	16′ 3
„ 26	2 4 29 N.	5 55·2 „	14 55	54 44	
Distance at 3 hours, 111° 33′ 34″; at 6 hours, 112° 57′ 16″.					
Apr. 20	11 23 54 N.	1 3·4 sub.	15 23·7	56 29·8	15 56
„ 21	11 44 26 N.	1 16·3 „	15 17·0	56 5·3	
Distance at noon, 68° 14′ 43″; at 3 hours, 69° 43′ 48″.					
May 19	19 41 37 N.	3 49·58 sub.	15 11·6	55 45·5	15 49
„ 20	19 54 26 N.	3 46·90 „	15 6·3	55 25·7	
Distance at noon, 61° 0′ 58″; at 3 hours, 62° 27′ 35″.					
Feb. 6	15 42 19 S.	14 22·51 add	14 55·0	54 44·3	16 11
„ 7	15 23 35 S.	14 26·16 „	14 59·0	54 59·1	
Distance at 0 hours, 57° 9′ 21″; at 3 hours, 58° 32′ 36″.					
Feb. 20	10 55 53 S.	14 1·7 add	16 4·4	58 59·2	16 11
„ 21	10 34 16 S.	13 54·7 „	16 9·6	59 18·0	
Distance at 3 hours, 100° 7′ 50″; at 6 hours, 101° 45′ 50″.					
Jan. 9	22 9 2 S.	7 19·4 add	14 55·8	54 47·2	16 17
„ 10	22 0 23 S.	7 44·1 „	15 0·2	55 3·6	
Distance at 0 hours, 76° 45′ 42″; at 3 hours, 78° 8′ 46″.					

Ship time obtained from moon's altitude.

When the sun or star is near the meridian, the ship mean time must be obtained by computing the hour angle of the moon and deducing from thence the ship mean time. This may be done by the following rule.

Rule LI.

Objects observed, moon and sun. Altitudes taken.
Ship mean time obtained from moon's altitude.

1. Get a Greenwich date.
2. Take out of Nautical Almanac and correct for Greenwich date the following quantities:—Right ascension of mean sun and sun's semidiameter; right ascension and declination of moon; semidiameter and horizontal parallax of moon.
3. Correct the sun's altitude for index correction, dip, semidiameter, and thus get the apparent altitude: from the apparent altitude subtract correction in altitude; the result is sun's true altitude, which subtract from 90° for sun's true zenith distance.
4. Correct the moon's altitude for index correction, dip, semidiameter (augmented); the result is the moon's apparent altitude. To the apparent altitude add the correction in altitude, the result subtract from 90° for the moon's true zenith distance.
5. When the moon's correction in altitude is taken out, take out also at the same opening of the book the auxiliary angle A.
6. Correct the observed distance for index and semidiameter.
7. *To find ship mean time.*

Under the moon's declination put the latitude of ship: take the difference if the names be alike, but their sum if

the names be unlike: under the result put the moon's zenith distance, and take the sum and difference. Add together the log. secants of the two first quantities in this form (rejecting the tens in index) and the halves of the log. haversines of the two last; the sum is the log. haversine of the moon's hour angle, to be taken out at the top of the page if the moon is west of the meridian, but at the bottom of the page if the moon is east of meridian. To the hour angle thus found add the moon's right ascension, and from right sum (increased if necessary by 24 hours) subtract the ascension of the mean sun; the remainder (rejecting 24 hours if greater than 24 hours) is ship mean time at the instant of observation.

8. Then proceed as in p. 211, arts. 8, 9.

EXAMPLES.

May 22, 1844, at 11^h 15^m A.M., mean time nearly, in lat. 50° 48′ N., and long. by account, 1° W., the following lunar observation was taken:—

	Obs. alt. sun's L. L.	Obs. alt. moon's L. L. E. of meridian.	Obs. dist. N. L.
	57° 53′ 0″	22° 53′ 2″	56° 26′ 6″
Index cor.	0 35 +	0 20 —	0 35 —

The height of eye above the sea was 24 feet; required the longitude.

Ship, May 21	23^h 15^m
Long. in time	4 W.
Greenwich, May 21 . .	23 19

Right ascen. mean sun.		Mcon's right ascen.		Moon's declination.	
21st	3^h 56^m $53^s·8$	23^h	7^h 55^m 24^s	23^h . . .	17° 5′ 12″ N.
23^h . . .	3 46·0	0	7 57 29	0 . . .	16 57 11 N.
19^m . . .	8·1		2 5		8 1
	4 0 42·9	·49940		·49940	
		1·46288		·87506	
Sun's semi. . 15′ 49″		1·96228	0 40	1·37446	2 32
			7 56 4		17 2 40 N.

	Moon's semi.			Moon's hor. par.	
21st, mid. . .	15′	1″·3	. . . .	55	7·6
22nd, noon .	15	5·7	. . . .	55	23·7
		4·4			16·1
Cor.		4·0	. . .		12·0
	15	5·3		55	19·6
Aug.		5·5			
	15	10·8			

Sun's altitude.

Obs. alt. .	57°	53′	0″
Index cor. .			35 +
	57	53	35
Dip . . .		4	49 —
	57	48	46
Semi. . . .		15	49
App. alt. .	58	4	35
Cor. in alt. .			36 —
	58	3	59
	90		
Sun's Z.D. .	31	54	1

Moon's altitude.

Obs. alt. .	22°	53′	2″
In. cor. . .		0	20 —
	22	52	42
Dip . . .		4	49 —
	22	47	53
		15	11
App. alt. .	23	3	4
Cor. in alt.		48	21
			17
	23	51	42
	90		
Zen. dist. .	66	8	18

Observed distance.

56°	26′	6″
		35 —
56	25	31
	15	11
	15	49
56	56	31
Aux. A.		
60	11	31
		4
		4
60	11	39

Moon's declination .	17°	2′	40″ N.	. Sec. . .	0·019510
Latitude	50	48	0 N.	. Sec. . .	0·199263
	33	45	20		
Moon's zen. dist. . .	66	8	18		
Sum	99	53	38	½ hav. .	4·883922
Difference	32	22	58	½ hav. .	4·445373
				hav. . .	9·548068

Hour angle	19^h	8^m	17^s
Moon's right ascen. . .	7	56	4
	27	24	21
Right asc. mean sun . .	4	0	45·5
Ship mean time	23	3	37·5

To find Greenwich mean time.

Zenith dist. . .	31°	56′	1″	Versines.				
Zenith dist. . .	66	8	18	40325 . .	91			
Sum	98	4	19 vers.	80612 . .	54			
				66003 . .	0			
App. alt. . . .	58°	4′	35″	56063 . .	43			
App. alt. . . .	5			01608 . .	2			
	81	7	39	44611 . .	191			
Aux. angle . .	60	11	39	190	True dist.			
Sum	141	19	18 vers.	44801 .	56	16	32	
Difference . .	20	56	0 vers.		55	15	36	at 21 hrs.
					56	41	20	
App. dist. . .	56°	56′	31″	47042	1	0	56	
Aux. angle . .	60	11	39	32212	1	25	44	
Sum	117	8	10 vers.	14830 .	2^h	7^m	56^s	
Difference . .	3	15	8 vers.		21			
				Greenwich mean time .	23	7	56	
				Ship mean time . . .	23	3	37·5	
				Long. in time		4	18·5	
				Longitude . .	1°	4′	37″ W.	

(201.) May 16, 1850, at 0^h 50^m P.M., mean time nearly, in lat. 42° 30′ N., and long. by account 29° 6′ W., the following lunar was taken :—

	Obs. alt. sun's L. L.	Obs. alt. moon's L. L. E. of meridian.	Obs. dist. N. L.
	61° 44′ 30″	39° 30′ 20″	63° 10′ 0″
Index cor.	2 10 —	1 10 —	0 20 +

The height of the eye above the sea was 20 feet; required the longitude. Ans., 29° 5′ 0″ W.

(202.) May 18, 1850, at 1^h 0^m P.M., mean time nearly, in lat. 43° N., and long. by account 41° 36′ W., the following lunar was taken :—

	Obs. alt. sun's L. L.	Obs. alt. moon's L. L. E. of meridian.	Obs. dist. N. L.
	64° 30′ 10″	18° 10′ 20″	90° 20′ 10″
Index cor.	2 10 +	1 10 —	1 20 +

The height of the eye above the sea was 15 feet; required the longitude. Ans., 41° 33′ 45″ W.

Elements from Nautical Almanac.

	Mean sun's R. A.	Moon's semi.	Hor. par.	Moon's R. A.	Moon's decl.	Sun's semi.
16th .	3^h 35^m $22^s{\cdot}14$	Noon, 16′ 16″·4	59′ 48″·0	7^h 58^m $16^s{\cdot}79$	18° 51′ 56″ N.	15′ 50″
		Mid. 16 13·9	59 34·0	8 0 48·01	18 47 43 N.	
Distance at noon, 61° 27′ 26″; at 3 hours, 63° 7′ 50″.						
18th .	3 43 15·25	Noon, 16 4·3	58 58·8	9 57 4·66	13 17 16 N.	15 49
		Mid. 16 0·6	58 45·3	9 59 23·58	13 8 8 N.	
Distance at 3 hours, 89° 31′ 39″; at 6 hours, 91° 9′ 1″.						

Rule LII.

Ship time obtained from star's altitude.

1. Get a Greenwich date.

2. Take out of the Nautical Almanac and correct for Greenwich date the following quantities:—star's right ascension and declination; right ascension of mean sun; moon's semidiameter and horizontal parallax.

3. Correct the star's observed altitude for index correction and dip; the result is the star's apparent altitude; from this subtract refraction; the remainder is the star's true altitude, which take from 90° to find star's true zenith distance.

4. Correct the moon's altitude for index correction, dip, semidiameter (augmented), and thus get the moon's apparent altitude; to this add the correction in altitude: the result is the moon's true altitude. Subtract the moon's true altitude from 90°; the remainder is the moon's true zenith distance.

5. When the correction of the moon's altitude is taken out, take out also at the same opening of the book the auxiliary angle A.

6. *To find ship mean time.*

Under star's declination put latitude of ship: take the sum if the names be unlike; but if the names be like take

the difference: under the result put the star's true zenith distance: take the sum and difference of the two last quantities put down.

Add together the log. secants of the two first quantities in this form (rejecting the tens in the index) and the halves of the log. haversines of the two last; the sum will be the log. haversine of the star's hour angle, to be taken out at the top of the page when the star is *west* of the meridian, and at bottom when east. To the hour angle thus found add the star's right ascension, and from the sum (increased if necessary by 24 hours) subtract the right ascension of the mean sun; the result (rejecting 24 hours if greater than 24 hours) will be ship mean time.

Then proceed as in p. 211, arts. 8, 9.

EXAMPLES.

June 2, 1849, at 10^h 17^m P.M., mean time nearly, in lat. 50° 50′ N., and long. by account 41° W., the following lunar observation was taken:—

	Obs. alt. Regulus W. of meridian.	Obs. alt. moon's L.L.	Dist. N. L.
	20° 21′ 40″	31° 11′ 0″	72° 36′ 30″
Index cor.	3 50 —	4 10 —	9 10 —

The height of the eye above the sea was 20 feet; required the longitude.

Ship, June 2	10^h	17^m
Long. in time	2	44 W.
Greenwich, June 2 . .	13	1

Right ascen. mean sun.				Moon's semi.	Hor. par.
2nd . .	4^h 48^m 21^s·2	Star's R. A. 10^h 0^m·20^s	2nd, mid.	14′ 49″	54′ 23
13^h .	2 8·1	Star's decl. 12 42 4 N.	3rd, noon	14 47	54 15
	4 45 29·3			2	8
			Cor. . .	0	0
				14 49	54 23
			Aug. .	7	
				14 56	

Star's altitude.		Moon's altitude.		Observed distance.	
Obs. alt. .	20° 21′ 40″	Obs. alt. .	31° 11′ 0″		72° 36′ 30″
Index cor. .	3 50 —	Index cor. .	4 10 —	Index cor. .	9 10 —
	20 17 50		31 6 50		72 27 20
Dip . . .	4 24 —	Dip . . .	4 24	Semi. . . .	14 56
App. alt. .	20 13 26		31 2 26	App. dist. .	72 42 16
Cor. in alt. .	2 37 —	Semi. . .	14 56		
	20 10 49	App. alt. .	31 17 22		Aux. ang. A.
	90	Cor. in alt.	44 34	. . . .	60 15 14
Zen. dist. .	69 49 11		20	. . . .	7
			32 2 16		60 15 21
			90		
		Zen. dist. .	57 57 44		

To find ship mean time.

Star's declination . .	12°	42′	4″ N.	. Sec. . .	0·010765
Latitude	50	50	0 N.	. Sec. . .	0·199573
	38	7	56		
Star's zenith dist. .	69	49	11		
Sum	107	57	7	. . ½ hav. .	4·907823
Difference	31	41	15	. . ½ hav. .	4·436186
				Hav. .	9·554344

Star's hour angle . . .	4^h	54^m	11^s
Star's right ascension .	10	0	20
	14	54	31
Right ascen. mean sun .	4	45	29
Ship mean time . . .	10	9	2

To find Greenwich mean time.

Star's zen. dist. .	69°	49′	11″
Moon's zen. dis.	57	57	44
	127	46	55 vers.
Star's app. alt. .	20°	13′	26″
Moon's app. alt.	31	17	22
	51	30	48
Aux. angle A. .	60	15	21
Sum	111	46	9 vers.
Difference . .	8	44	33 vers.

Versines.				
81360 . .	131			
23453 . .	56			
12447 . .	212			
70828 . .	41			
11594 . .	24			
99682 . .	464			
464				
00146 .		72°	33′	4″
127		72	19	35 at 12 hours
19		73	49	27 at 15 hours
1·12548			13	29
·30167		1	29	52
·82381 .		0^h	27^m	0^s
		12		
Greenwich mean time .		12	27	0
Ship mean time . . .		10	9	2
Long. in time		2	17	58

Longitude . 34° 29′ 30″ W.

(203.) Jan. 8, 1851, at 7^h 0^m P.M., mean time nearly, in lat. 50° 40′ N., and long. by account 4° E., the following lunar was taken:—

	Obs. alt. Aldebaran E. of meridian.	Obs. alt. moon's L. L.	Obs. dist. F. L.
	45° 20′ 10″	30° 30′ 0″	71° 31′ 10″
Index cor.	2 20 —	2 10 +	3 30 —

The height of eye above the sea was 18 feet; required the longitude. Ans., 3° 46′ 30″ E.

(204.) Jan. 9, 1851, at 7^h 50^m P.M., mean time nearly, in lat. 49° 40′ N., and long. by account 10° E., the following lunar was taken:—

	Obs. alt. Pollux E. of meridian.	Obs. alt. moon's L. L.	Obs. dist. F. L.
	37° 10′ 10″	31° 50′ 10″	103° 20′ 0″
Index cor.	1 10 —	1 20 +	1 30 +

The height of eye above the sea was 18 feet; required the longitude. Ans., 10° 19′ 15″ E.

(205.) April 18, 1850, at 9^h 40^m P.M., mean time nearly, in lat. 56° 10′ N., and long. by account 23° E., the following lunar was taken:—

	Obs. alt. α Virginis E. of meridian.	Obs. alt. moon's L. L.	Obs. dist. F. L.
	19° 40′ 0″	48° 40′ 20″	91° 30′ 20″
Index cor.	1 10 +	1 20 +	0 30 —

The height of eye above the sea was 20 feet; required the longitude. Ans., 23° 3′ 30″ E.

(206.) April 17, 1850, at 8^h 45^m P.M., mean time nearly, in lat. 51° 20′ N., and long. by account 5° 10′ E., the following lunar was taken:—

	Obs. alt. α Virginis E. of meridian.	Obs. alt. moon's L. L.	Obs. dist. F. L.
	18° 15′ 30″	36° 25′ 10″	105° 33′ 28″
Index cor.	1 10 +	1 20 —	0 20 +

The height of eye above the sea was 20 feet; required the longitude. Ans., 5° 8′ E.

(207.) November 17, 1847, at 2^h 50^m A.M., mean time nearly, in lat. 44° 30′ N., and long. by account 121° E., the following lunar was taken:—

	Obs. alt. α Arietis W. of meridian.	Obs. alt. moon's L. L.	Obs. dist. N. L.
	29° 58′ 10″	32° 20′ 30″	58° 30′ 20″
Index cor.	5 30 +	2 10 —	2 15 —

The height of eye above the sea was 20 feet; required the longitude. Ans., 121° 5′ 45″ E.

(208.) March 20, 1845, at 7^h 50^m P.M., mean time nearly, in lat. 49° 50′ N., and long. by account 1° 30′ E., the following lunar was taken:—

	Obs. alt. Aldebaran W. of meridian.	Obs. alt. moon's L. L.	Obs. dist. N. L.
	37° 4′ 30″	38° 17′ 20″	75° 15′ 10″
Index cor.	2 4 —	1 5 +	0 20 +

The height of eye above the sea was 20 feet; required the longitude. Ans., 1° 34′ 30″ E.

Elements from Nautical Almanac.

	Right ascen. mean sun.		Moon's semi.	Hor. par.		
Jan. 8 .	19h 9m 45s·72 .	Noon	14′ 48″·8 .	54′ 21″·8	Star's R. A. .	4h 27m 22s·7
		Mid.	14 52 ·0 .	54 33 ·3	Star's decl. .	16° 12′ 13″ N.
	Distance at 6 hours, 71° 1′ 57″; at 9 hours, 69° 32′ 16″.					
Jan. 9 .	19 13 42 ·27 .	Noon	14 55 ·8 .	54 47 ·2	Star's R. A. .	7h 36m 12s
		Mid.	15 0 ·2 .	55 3 ·6	Star's decl. .	28° 22′ 46″ N.
	Distance at 6 hours, 103° 8′ 4″; at 9 hours, 101° 37′ 51″.					
April 18	1 44 58 ·62 .	Noon	16 8 ·8 .	59 15 ·4	Star's R. A. .	13h 17m 19s
		Mid.	16 8 ·7 .	59 14 ·8	Star's decl. .	10° 22′ 43″ S.
	Distance at 6 hours, 92° 10′ 18″; at 9 hours, 90° 24′ 32″.					
April 17	1 41 2 ·07 .	Noon	16 8 ·2 .	59 13 ·1	Star's R. A. .	13h 17m 19s
		Mid.	16 8 ·7 .	59 14 ·9	Star's decl. .	10° 22′ 43″ S.
	Distance at 6 hours, 106° 16′ 11″; at 9 hours, 104° 30′ 26″.					
Nov. 16 .	15 39 44 ·50 .	Noon	16 2 ·1 .	58 50 ·9	Star's R. A. .	1h 58m 38s
		Mid.	16 7 ·5 .	59 10 ·6	Star's decl. .	22° 44′ 30″ N.
	Distance at 21 hours, 50° 27′ 26″; at 0 hours, 48° 44′ 36″.					
Mar. 20	23 51 30 ·11 .	Noon	15 12 ·2 .	55 47 ·6	Star's R. A. .	4h 27m 8s
		Mid.	15 17 ·4 .	56 6 ·7	Star's decl. .	16° 11′ 30″ N.
	Distance at 6 hours, 74° 7′ 12″; at 9 hours, 75° 42′ 9″.					

Rule LIII.

Ship mean time obtained from moon's altitude.

Objects observed, moon and star.

This differs very little from Rule LI., p. 217.

1. Get a Greenwich date.

2. Take out of the Nautical Almanac and correct for Greenwich date the following quantities :—Right ascension of mean sun; moon's right ascension and declination; semidiameter, and horizontal parallax.

3. Correct the star's altitude for index correction, dip. and thus get the apparent altitude: from the star's apparent altitude subtract refraction; the result is the true altitude, which take from 90° for the star's true zenith distance.

Then proceed as in 4, 5, &c., p. 217.

Rule LIV.

Ship mean time from planet's altitude.

Objects observed, moon and planet.

1. Get a Greenwich date.

2. Take out of the Nautical Almanac and correct for Greenwich date the following quantities:—Right ascension of mean sun; planet's right ascension and declination; planet's horizontal parallax (if great accuracy is required); moon's semidiameter and horizontal parallax.

3. Correct the planet's observed altitude for index correction and dip, and thus get the apparent altitude; from the apparent altitude subtract the refraction and add the parallax in altitude (usually neglected, being very small); the result is the planet's true altitude, which subtract from 90° to get the true zenith distance.

4. Correct the moon's altitude as in 4, p. 221.

5. Get the auxiliary angle A as in 5, p. 221.

6. To find ship mean time as in 6, p. 221, using planet's declination and right ascension instead of star's.

7. Then proceed as in arts. 8, 9, p. 211.

EXAMPLE.

September 24, 1849, at 7^h 50^m P.M., mean time nearly, in lat. 47° 50′ N., and long. by account 2° 30′ W., the following lunar observation was taken:—

	Obs. alt. Saturn E. of meridian.	Obs. alt. moon's L. L.	Obs. dist. F. L.
	16° 55′ 0″	15° 30′ 45″	89° 51′ 36″
Index cor.	3 5 —	3 10 —	3 0 +

The height of eye above the sea was 20 feet; required the longitude.

Ship, Sept. 24	7^h	50^m
Long. in time		10 W.
Greenwich, Sept. 24 . .	8	0

Right ascen. mean sun.		Planet's right ascen. and declination.			
24th . . .	$12^h\ 12^m\ 48^s{\cdot}4$	24th . .	$0^h\ 21^m\ 50^s{\cdot}2$	. . . . . .	$0^\circ\ 31'\ 23''$ S.
5^h . .	1 18·8	25th . . .	0 21 33·0	. . . .	0 33 17 S.
	12 14 7·2		17·2		1 54
		Cor. . . .	6·0	Cor. . .	38
Planet's hor. par. $1''{\cdot}0$.			0 21 44		0 32 1 S.

		Moon's hor. par.		Moon's semi.	
Obs. dist. .	$89^\circ\ 51'\ 36''$	Noon	$54'\ 13''{\cdot}6$	. . . .	$14'\ 46''{\cdot}6$
Index cor. .	3 0 +	Mid.	54 16·2	. . . .	14 47·3
	89 54 36		2·6		0·7
Semi. . . .	14 51 —	Cor. . .	1·7	Cor. . . .	0·5
App. dist. .	89 39 45		54 15·3		14 47·1
				Aug. . .	3·9
					14 51

Planet's altitude.		Moon's altitude.		
Obs. alt. . .	$16^\circ\ 55'\ 0''$	Obs. alt. .	$15^\circ\ 30'\ 45''$	
In. cor. . .	3 5 —	In. cor. . .	3 10 —	
	16 51 55		15 27 35	
Dip . . .	4 24 —	Dip . . .	4 24 —	
App. alt. . .	16 47 31		15 23 11	
Refr. 3 11 —		Semi. . .	14 51 +	
Par. 1 +	3 10 —		15 38 2	Angle A.
	16 44 21	Cor. in alt. .	48 36 . .	$60^\circ\ 7'\ 29''$
	90		14	2
Zen. dist. .	73 15 39		16 26 52	0
			90	60 7 31
		Zen. dist. .	73 33 8	

To find ship mean time.

Latitude	$47^\circ\ 50'\ 0''$ N.	. Sec. . .	0·173090
Declination . . .	0 32 1 S.	. Sec. . .	0·000019
	48 22 1		
Zenith distance . .	73 15 39		
Sum	121 37 40	½ hav. .	4·941037
Difference	24 53 38	½ hav. .	4·333480
		Hav. .	9·447626
		Hour angle	$19^h\ 44^m\ 16^s$
		Planet's right ascen. . .	0 21 44
			20 6 0
		Right ascen. mean sun .	12 14 7·2
		Ship mean time . . .	7 51 52·8

To find Greenwich mean time.

Zenith dist. . .	73°	15′	39″	Versines.						
Zenith dist. . .	73	33	8	36764	. .	126				
	146	48	47 vers.	44491	. .	19				
				14471	. .	130				
App. alt. . . .	16°	47′	31″	64128	. .	39				
App. alt. . . .	15	38	2	29931	. .	33				
	32	25	33	89985	. .	347				
Aux. angle . .	60	7	31	347						
Sum	92	33	4 vers.	90132	.		89°	26′	5″	
Difference . .	27	41	58 vers.	110			90	26	30	at 6 hours
				22			88	57	4	
App. dist. . .	89°	39′	45″	·47412			1	0	25	
Aux. angle . .	60	7	31	·30377			1	29	26	
Sum	149	47	16 vers.	·17035			2h	1m	36s	
Difference . .	29	32	14 vers.				6			
				Greenwich mean time	.		8	1	36	
				Ship mean time	. . . .		7	51	53	
				Long. in time	. . . .			9	43 W.	

Longitude . . 2° 25′ 45″ W.

Longitude by lunar.—Altitudes calculated.

To find ship mean time.

The error of the chronometer on ship mean time is found a little before or after the lunar distance is taken. For this purpose the observer selects any heavenly body whose bearing is nearly east or west, so that the error in the altitude may produce the smallest error in the resulting hour angle (see Rule XLIII.) Then the time being noted by the same chronometer when the distance is taken, ship mean time is known at the same instant, by applying the error found by the above observation.

Rule LV.

Objects observed, moon and stars.

1. Get a Greenwich date.

2. Take from the Nautical Almanac and correct for the Greenwich date, the following quantities: sun's declination, equation of time and semidiameter, right ascension of mean sun. Moon's right ascension and declination, moon's semidiameter and horizontal parallax.

3. *To find the sun's hour angle.*

To the time shown by chronometer at the observation apply the error of chronometer with its proper sign, and thus get ship mean time; to this apply equation of time, the result is ship apparent time, and also the sun's hour angle.

4. *To calculate the sun's altitude.*

Under the latitude* put down the sun's declination, take the sum if the names be unlike, but the difference if the names be alike; call the result v; add together constant log. 6·301030, log. cos. latitude, log. cos. sun's declination, and log. haversines sun's hour angle, reject 30 in the index, and look out the result as a logarithm, and take its natural number to the nearest unit.

Add together this natural number and the versine of the quantity v found above: the sum is the versine of the sun's *true zenith distance*, which find in the tables and subtract from 90°: the result is the sun's true altitude.

To find the sun's apparent altitude.

To the true altitude just found, *add* correction in altitude (for parallax and refraction) the result will be the sun's apparent altitude very nearly.†

* When great accuracy is required, the latitude and horizontal parallax should be corrected for the spheroidal figure of the earth.

† In strictness the table for correction of altitude ought to have been entered with the apparent altitude, instead of the true, to get the correction in altitude; but the above is sufficiently correct.

5. *To find the moon's hour angle.*

To right ascension of mean sun add ship mean time, and from the sum (increased if necessary by 24 hours), subtract the moon's right ascension; the result is the moon's hour angle (rejecting 24 hours, if greater than 24 hours).

6. *To find the moon's true altitude.*

Under the latitude put the moon's declination, take the sum if the names be unlike, and the difference if the names be alike, call the result $v_{,}$. Add together the constant quantity, 6·301030, log. cos. latitude, log. cos. moon's declination, and log. haversine of moon's hour angle, reject 30 from the index, and look out the result as a logarithm, and take its natural number. To this natural number add the versine of the quantity $v_{,}$ found as above; the sum is the versine of the moon's true zenith distance, which find in the table and subtract from 90°; the result is the moon's true altitude.

To find the moon's apparent altitude.

Consider the moon's true altitude just found as the apparent altitude; enter the table with it, and take out the correction in altitude thus approximately; subtract this correction from the moon's true altitude, and thus get the moon's apparent altitude nearly. Then enter the table again with this corrected altitude, and thus take out the correction in altitude more exactly; subtract this correction from the moon's *true* altitude, and the result will be the moon's apparent altitude very nearly.

Take out at the same opening of the table the auxiliary angle A.

Correct the moon's semidiameter for augmentation. Then proceed as in arts. 6, 8, 9, p. 211.

EXAMPLE.

August 19, 1843, in lat. 50° 48′ N., and long. by account 1° 6′ W., when a chronometer showed $11^h\ 10^m\ 19^s{\cdot}8$ A.M.,

the observed distance of the nearest limb of the sun and moon was 76° 51′ 26″, the error of the chronometer on ship mean time being fast 7^m $29^s{\cdot}3$, and the index correction 1′ 55″ + ; required the longitude.

Time by chro. . .	11^h	10^m	$19^s{\cdot}8$
Error of chro. . . .		7	29·3 fast
Ship mean time . .	23	2	50·5
Long. in time . . .		4	24·0 W.
Green. date, Aug. 18	23	7	
Ship mean time . .	23^h	2^m	$50^s{\cdot}5$
Equa. of time . . .		3	30·6 sub.
Sun's hour angle . .	22	59	19·9

	Sun's declination.					Eq. of time.	
18th .	13°	15′	12″N. . . .			3^m	$43^s{\cdot}2$ sub.
19th .	12	55	47 N. . . .			3	30·1
		40	35				13·1
·01629				·01629			
·64692				2·91615			
·66321		39	5	2·93244			12·6
	13	54	17 N.			3	30·6

Sun's semi. . 15′ 49″

Moon's right ascen.			
18th at 23^h . .	4^h	27^m	27^s
„ 24 . .	4	29	41
		2	14
·93305			
1·42920			
2·36225		0	16
	4	27	43

Right ascen. mean sun.			
18th . . .	9^h	44^m	$48^s{\cdot}22$
		3	36·84
			1·14
	9	48	26·20
Ship M.T.	23	2	50·50
	32	51	16·70
M.'s R. A.	4	27	43·0
M.'s H. A.	4	23	33·70

Moon's declin.			
	23°	43′	13″ N.
	23	43	27 N.
		0	14
·93305			
2·41018			
3·34323		0	2
	23	43	15 N.

	Moon's semi.			Moon's hor. par.		
18th, at mid. . .	14′	59″·7			55′	1″·8
19th, at noon . .	15	4·5			55	19·1
		4·8				17·3
·03321				·03321		
3·35218				2·79538		
3·38539		4·4		2·82859		16·0
	15	4·1			55	17·8
Aug.		8·2				
Moon's semi. . .	15	12·3				

To calculate sun's altitude.				*To calculate moon's altitude.*			
Latitude . . .	50°	48′	0″ N.	Latitude . . .	50°	48′	0″ N.
Sun's declination	13	54	17 N.	Moon's dec. . .	23	43	15 N.
	36	53	43 v.		27	4	45 $v_{,}$
Const. log.			6·301030	Const. log..			6·301030
Cos. lat.			9·800737	Cos. lat.			9·800737
Cos. sun's dec.. . . .			9·987085	Cos. moon's dec. . .			9·961832
Hav. sun's hour angle .			8·240938	Hav. moon's hour ang.			9·470952
Log.			4·329790	Log..			5·534551
Nat. No..			21369	Nat. No.			342413
Vers. v.			0200141	Vers. $v_{,}$.			0109522
			124				99
Vers. sun's zen. dist.. .			0221634	Vers. moon's zen. dist.			0452034
			574				1950
			60				84
Sun's zenith dist. .	38°	53′	20″	Moon's zenith dist..	56°	46′	21
	90				90		
Sun's true alt. . .	51	6	40	Moon's true alt.. .	33	13	39
Cor. in alt. . . .		0	41	Cor. in alt. . . .		43	42
Sun's app. alt. . .	51	7	21	M.'s app. alt. nearly	32	29	57
						44′	53″
Aux. angle A.							10
	60° 16′ 4″			True cor. in alt. .		45	3
	4			Moon's true alt. .	33	13	39
	3			Moon's app. alt. .	32	28	36
	60 16 11 Arc A.						

Observed distance . . .	76°	51′	26″
Sun's semi..		15	49
Moon's semi.		15	12
Apparent distance . .	77	24	22

To find Greenwich mean time.

Sun's zenith dist. .	38°	58′	20″	
Moon's zenith dist.	56	46	21	
Sum	95	39	41	*v.*
Sun's app. alt... .	51	7	21	
Moon's app. alt. .	32	28	36	
Sum	83	35	57	
Arc. A.	60	16	11	
Sum	143	52	8	*u.*
Difference. . . .	23	19	46	*v.*
App. dist. . . .	77	24	22	
Arc A.	60	16	11	
Sum	137	40	33	*v.*
Difference . . .	17	8	11	*u.*

Versines.		
98451	.	198
07647	.	23
81669	.	89
39239	.	108
44378	.	16
71384	.	434
434		
71818	. . .	
649		
169		

	True dist.			
	76°	48′	36″	
	77	48	59	at 21h
	76	24	3	
·47436 .	1	0	23	
·32619 .	1	24	56	
·14817 .	2h	7m	58s	
	21			
Greenwich mean time .	23	7	58	
Ship mean time.	23	2	50	
Longitude in time. . .		5	8	

Longitude . 1° 17′ W.

(209.) September 16, 1843, in latitude 50° 48′ N., and long. by account, 1° 6′ W., when a chronometer showed $9^h\ 34^m\ 6^s{\cdot}6$ A.M., the observed distance of the nearest limb of the sun and moon was 96° 26′ 18″, the index correction being + 1′ 32″, and the error of chronometer on ship mean time being fast $7^m\ 59^s$, required the longitude.

Ans., 1° 35′ 30″ W.

(210.) October 14, 1843, in lat. 50° 48′ N., and long. by account 1° 6′ W., when a chronometer showed $9^h\ 53^m\ 57^s{\cdot}1$, A.M., the observed distance of the nearest limb of the sun and moon was 114° 58′ 22″, the error of the chronometer being slow $3^m\ 27^s$, and the index correction + 1′ 32″; required the longitude. Ans., 1° 30′ 30″ W.

(211.) October 16, 1843, in lat. 50° 48′ N., and long. by account 1° 6′ W., when a chronometer showed $9^h\ 58^m\ 9^s{\cdot}8$ A.M., the observed distance of the nearest limb of the sun

and moon was 91° 45′ 38″, the error of the chronometer being slow $3^m\ 26^s{\cdot}5$, and the index correction + 1′ 30″ required the longitude. Ans., 1° 31′ 30″ W.

(212.) August 17, 1843, in lat. 50° 37′ 30″ N., and long. by account 1° 6′ W., when a chronometer showed $10^h\ 42^m\ 28^s{\cdot}7$ A.M., the observed distance of the nearest limb of the sun and moon was 99° 22′ 35″, the error of the chronometer on ship mean time being fast $7^m\ 44^s$, and the index correction + 1′ 55″; required the longitude.

Ans., 1° 17′ W.

(213.) May 25, 1843, in lat. 50° 48′ N., and long. by account 1° 6′ W., when a chronometer showed $11^h\ 19^m\ 15^s{\cdot}1$ A.M., the observed distance of the nearest limb of the sun and moon was 42° 48′ 48″·3, the error of the chronometer on ship mean time being slow $3^m\ 29^s{\cdot}7$, and the index correction + 3′ 30; required the longitude.

Ans., 1° 5′ 45″ W.

(214.) May 25, 1843, in lat. 50° 37′ 30″ N., and long. by account 1° 6′ W., when a chronometer showed $11^h\ 4^m\ 12^s{\cdot}2$ A.M., the observed distance of the sun and moon's nearest limb was 43° 1′ 3″, the error of chronometer on ship mean time being fast $12^m\ 53^s{\cdot}5$, and the index correction + 0′ 57″; required the longitude. Ans., 1° 33′ W.

Elements from Nautical Almanac.

	Sun's declination.	Equation of time.		Mean sun's right ascen.
Sept. 15	3° 11′ 23″ N.	$4^m\ 42^s{\cdot}5$ add	15th	$11^h\ 35^m\ 11^s{\cdot}72$
„ 16	2 48 15 N.	5 3·6		

	Moon's right ascen.	Moon's declin.	Moon's semi.		Hor. par.
15th, at 21^h	$4^h\ 59^m\ 11^s{\cdot}7$	23° 46′ 38″·6	Mid.	14′ 58″·2	54′ 56″·2
„ 22^h	5 1 25·7	23 47 9·0	Noon	15 2·8	55 13·1

Distance at 21 hours, 96° 42′ 14″; at noon, 95° 17′ 50″.

	Sun's declination.	Equation of time.		Mean sun's right ascen.
Oct. 13	7° 38′ 12″ S.	$13^m\ 36^s{\cdot}0$ add	13th	$13^h\ 25^m\ 35^s{\cdot}20$
„ 14	8 0 40 S.	13 50·2		

	Moon's right ascen.	Moon's declin.	Moon's semi.		Hor. par.
13th, at 22^h	$5^h\ 40^m\ 1^s{\cdot}1$	23° 21′ 26″ N.	Mid.	14′ 57″·8	54′ 54″·7
„ 23^h	5 42 14·6	23 19 43 N.	Noon	15 2·1	55 10·3

Distance at 22 hours, 115° 27′ 42″; at noon, 114° 3′ 26″.

	Sun's declination.	Equation of time.	Mean sun's right ascen.
Oct. 15	8° 23′ 1″·4 S.	14m 3s·8 add	15th . 13h 33m 28s·31
„ 16	8 45 15 ·6 S.	14 16·8	

	Moon's right ascen.	Moon's declin.	Moon's semi.	Hor. par.
15th, at 22h	7h 26m 58s·8	19° 45′ 6″ N.	Mid. . 15′ 18″·0	56′ 8″·7
„ 23h	7 29 11 ·9	19 37 45 N.	Noon . 15 24 ·3	56 32 ·1

Distance at 21 hours, 92° 27′ 48″; at noon, 90° 58′ 57″.

	Sun's declination.	Equation of time.	Mean sun's right ascen.
Oct. 16	13° 53′ 24″ N.	4m 7s·9 sub.	16th . 9h 36m 55s·12
„ 17	13 34 24 N.	3 55 ·8	

	Moon's right ascen.	Moon's declin.	Moon's semi.	Hor. par.
16th, at 22h	2h 42m 36s	19h 40m 25s N.	Mid. . 14′ 47″·5	54′ 16″·8
„ 23h	2 44 29	19 47 38 N.	Noon . 14 49 ·5	54 24 ·3

Distance at 21 hours, 99° 59′ 6″; at noon, 98° 37′ 11″.

	Sun's declination.	Equation of time.	Mean sun's right ascen.
Oct. 24	20° 42′ 10″·8 N.	3m 30s·9 add	24th . 4h 5m 44s·32
„ 25	20 53 16 ·6 N.	3 25 ·6	

	Moon's right ascen.	Moon's declin.	Moon's semi.	Hor. par.
24th, at 23h	1h 8m 43s·11	12° 39′ 15″·8	Mid. . 14′ 44″·8	54′ 6″·8
„ 24h	1 10 36 ·96	12 49 47 ·9	Noon . 14 45 ·7	54 10 ·3

Distance at 21 hours, 44° 0′ 59″; at 0 hours, 42° 39′ 15″.

Rule LVI.

Longitude by lunar.—Altitudes calculated.

Objects observed, moon and star.

1. Get a Greenwich date.

2. Take from the Nautical Almanac, and correct for the Greenwich date, the following quantities: star's declination and star's right ascension, right ascension of mean sun, moon's right ascension and moon's declination; moon's semidiameter and moon's horizontal parallax.

3. *To find star's hour angle.*

To the right ascension of mean sun add mean time at ship,* and from the sum subtract the star's right ascension; the remainder is the hour angle of the star.

* Ship mean time is found by an altitude of a heavenly body taken a little before or after the lunar as directed, p. 229.

4. *To find the moon's hour angle.*

From the same sum (viz. right ascension of mean sun and ship mean time) subtract the moon's right ascension; the remainder is the hour angle of the moon.

5. *To calculate the star's true altitude.*

Proceed as in 4, p. 230, using the star's declination instead of the sun's.

To find star's apparent altitude.

To the true altitude just found add the refraction; the result is the star's apparent altitude.

6. *To find the moon's true and apparent altitude* proceed as in 6, p. 231.

7. Proceed then as in former rule (arts. 6, 8, 9, p. 211).

EXAMPLE.

May 16, 1842, in lat. 50° 37′ 30″ N., and long. by account 1° 6′ W., when a chronometer showed $10^h\ 39^m\ 26^s$ P.M., the observed distance of the star α Virginis from the moon's farthest limb was 64° 1′ 50″, index correction + 0′ 40″, the error of the chronometer on ship mean time being fast $4^m\ 14^s$; required the longitude.

Time by chro.	10^h	39^m	26^s
Error of chro.		4	14
Ship m. time .	10	35	12
Long. in time		4	24
Gr. May 16 .	10	39	36

Star's declination.
10° 20′ 25″ S.

Star's right ascen.

	13^h	16^m	$55^s{\cdot}70$
	14	12	6 ·05
Star's hour angle . .	0	55	10 ·35

Right ascen. mean sun.

16th . . .	3^h	35^m	$9^s{\cdot}00$
		1	38 ·56
			6 ·40
			·09
	3	36	54 ·05
	10	35	12 ·00
	14	12	6 ·0
	9	19	39 ·9
Moon's hour angle .	4	52	26 ·07

Moon's right ascen.

16th, at 10^h		9^h	18^m	$9^s{\cdot}98$
,, 11^h		9	20	24 ·48
			2	14 ·50
·18046				
1·42920				
1·60966			1	30 ·00
		9	19	39· 98

Moon's declin.

	13°	25′	13″ N.
	13	11	47 N.
		13	26
·18046			
·64997			
·83043		8	52
	13	16	21 N.

Moon's semi.

Noon .	16′	7″·0
Mid. .	16	8 ·1
		1 ·1
·05183		
3·99203		
4·04386		1 ·0
	16	8 ·0
Aug.		5 ·8
	16	13 ·8

Hor. par.

	59′	8″
	59	12 ·6
		4 ·1
·05183		
3·42064		
3·47247		3 ·6
	59	12 ·1

To calculate star's altitude.					*To calculate moon's altitude.*				
Latitude . . .	50°	37′	30″	N.	Latitude . . .	50°	37′	30″	N.
Star's declination	10	20	25	S.	Moon's dec. . .	13	16	21	N.
	60	57	55	v.		37	21	9	$v_{\prime}$
Const log.			6·301030		Const. log..			6·301030	
Cos. lat.			9·802359		Cos. lat.			9·802359	
Cos. star's dec. . . .			9·992887		Cos. moon's dec.. . .			9·988095	
Hav. star's hour angle .			8·158830		Hav. moon's hour ang.			9·549884	
Log.			4·255106		Log.			5·641368	
Nat. No..			17993		Nat. No.			437892	
Versine v.			0514427		Vers. $v_{\prime}$.			0205056	
			232					26	
Vers. star's zen. dist. .			0532652		Vers. moon's zen. dist.			0642974	
Star's zen. dist. . .	62°	8′	16″		Moon's zenith dist.	69°	4′	56″	
	90					90			
Star's true alt. . .	27	51	44		M's. tr. alt. (nearly)	20	55	4	
Cor. in alt. . . .	+	1	50		Cor. in alt.. . . .	—	52	36	
Star's app. alt. . .	27	53	34		Ap. alt. (nearly) .	20	2	28″	

Aux. Ang. A.						
					52′	48″
60° 10′ 47″						5
1		True cor. in lat. .			52	53
0		Moon's true alt.. .		20	55	4
60 10 48 Arc A.		Moon's app. alt. . .		20	2	11

Observed distance . .	64°	1′	50″
Index correction. . .		0	40 +
	64	2	30
Moon's semidiameter .		16	14
Apparent distance . .	63	46	16

To find Greenwich mean time.

				Versines.		True dist.		
Star's zenith dist. .	62°	8′	16″					
Moon's zenith dist.	69	4	56	58908 . 44				
Sum	131	13	12 v.	10676 . 152				
				22769 . 3				
Star's app. alt. . .	27	53	34	58469 . 16				
Moon's app. alt. .	20	2	11	01955 . 8				
Sum	47	55	45	52777 . 223				
Arc A	60	10	48	223				
Sum	108	6	33 v.	53000 . . .		63°	26′	55″
Difference . . .	12	15	3 v.	761		64	24	13 at 9^h
				239		62	38	17
App. dist. . . .	63	46	16		·49712 .	0	57	18
Arc A	60	10	48		·23024 .	1	45	56
Sum	123	57	4 v.		·26688 .	1^h	37^m	22^s
Difference . . .	3	35	28 v.			9		
				Greenwich mean time . .		10	37	22
				Ship mean time		10	35	12
				Longitude in time . . .			2	10

Longitude . 0° 32′ 30″ W.

(215.) April 20, 1847, in lat. 50° 37′ 12″ N., and long. by account 1° 6′ W., when a chronometer showed $8^h\ 58^m\ 45^s$ P.M., the observed distance of the star α Leonis from the moon's farthest limb was 46° 2′ 12″, index correction + 0′ 30″, the error of chronometer being fast $3^m\ 22^s$; required the longitude. Ans., 0° 55′ 45″ W.

(216.) December 10, 1845, in lat. 50° 37′ 30″ N., and long. by account 1° 6′ W., when a chronometer showed $9^h\ 24^m\ 48^s{\cdot}3$ P.M., the observed distance of the star Pollux from the moon's farthest limb was 65° 28′ 30″, index correction + 0′ 30″, the error of the chronometer on ship mean time being fast $12^m\ 50^s{\cdot}8$; required the longitude.

Ans., 0° 44′ 15″ W.

(217.) April 19, 1847, in lat. 50° 48′ N., and long. by

account 1° 6′ W., when a chronometer showed $8^h\ 40^m\ 18^s$ P.M., the observed distance of the star Regulus from the moon's farthest limb was 59° 11′ 1″·6, index correction + 30″, the error of the chronometer on ship mean time being fast $9^m\ 30^s{\cdot}4$; required the longitude.

Ans., 1° 7′ W.

(218.) September 1, 1843, in lat. 50° 37′ 30″ N. and long. by account, 1° 6′ W., when a chronometer showed $8^h\ 2^m\ 54^s{\cdot}4$ P.M., the observed distance of the planet Jupiter from the moon's farthest limb was 64° 19′ 57″, index correction + 1′ 50″ the error of the chronometer on ship mean time being fast $2^m\ 2^s{\cdot}6$; required the longitude.

Ans., 0° 35′ 15″ W.

(219.) September 5, 1843, in lat. 50′ 48″ N., and long. by account 1° 6′ W., when a chronometer showed 8° 52′ 39″ P.M., the observed distance of the planet Mars from the moon's nearest limb was 45° 11′ 23″·3, index correction + 1′ 50″, the error of the chronometer being fast $4^m\ 47^s{\cdot}4$; required the longitude. Ans., 1° 19′ 45″ W.

Elements from Nautical Almanac.

	Star's declin.	Star's right ascen.	Mean sun's right asc.
April 20	12° 42′ 30″ N.	$10^h\ 0^m\ 15^s{\cdot}35$	$1^h\ 51^m\ 48^s{\cdot}09$

	Moon's right ascen.	Moon's declin.	Moon's semi.	Hor. par.
20th, at 8^h	$6^h\ 51^m\ 3^s{\cdot}8$	17° 39′ 51″ N.	Noon . 15′ 23″·7	56′ 29″·8
9^h	6 53 16·8	17° 36 40 N.	Mid. . 15 17·0	56 5·3

Distance at 6 hours, 46° 51′ 27″; at 9 hours, 45° 16′ 23″.

	Star's declin.	Star's right ascen.	Mean sun's right asc.
10th	28° 23′ 22″ N.	$7^h\ 35^m\ 54^s{\cdot}9$	$17^h\ 16^m\ 17^s{\cdot}09$

	Moon's right ascen.	Moon's declin.	Moon's semi.	Hor. par.
10th, at 9^h	$2^h\ 54^m\ 19^s{\cdot}38$	16° 40′ 52″ N.	Noon . 15′ 11″·5	55′ 45″·0
„ 10^h	2 56 27·74	16 47 6 N.	Mid . 15 7·7	55 30·9

Distance at 9 hours, 65° 12′ 56″; at mid., 63° 41′ 12″.

	Star's declin.	Star's right ascen.	Mean sun's right asc.
19th	12° 42′ 30″ N.	$10^h\ 0^m\ 15^s{\cdot}35$	$1^h\ 47^m\ 51^s{\cdot}54$

	Moon's right ascen.	Moon's declin.	Moon's semi.	Hor. par.
19th at 8^h	5^h 56′ $41^s{\cdot}12$	18° 28′ 9″·6 N.	Noon . 15′ 38″·4	57′ 23″·5
„ 9^h	5 58 59·68	18 27 17·2 N.	Mid. . 15′ 30·9	56 56·1

Distance at 6 hours, 59° 46′ 25″; at 9 hours, 58° 8′ 6″.

	Star's declin.	Star's right ascen.	Mean sun's right asc.
1st	15° 46′ 6″·8 S.	21h 33m 3s·13	10h 39m 59s·98
2nd	15 48 22·9 S.	21 32 35·33	

	Moon's right asc.	Moon's declin.	Moon's semi.	Hor. par.
1st, at 8h	17h 0m 32s·9	23° 56′ 5″·7 S.	Noon. 15′ 54″·8	58′ 24″·0
„ 9h	17 3 2·2	23 56 34·4 S.	Mid. 15 49·5	58 4·5

Distance at 6 hours, 65° 7′ 44″; at 9 hours, 63° 24′ 57″.

	Star's declin.	Star's right ascen.	Mean sun's right asc.
5th	26° 34′ 9″·5 S	17h 32m 5s·8	10h 55m 46s·19
6th	26 34 29·5 S.	17 34 27·8	

	Moon's right ascen.	Moon's declin.	Moon's semi.	Hor. par.
5th, at 8h	20h 41m 16s·9	15° 1′ 2″ S.	Noon. 15′ 15″·7	56′ 0″·4
„ 9h	20 43 20·7	14 50 47 S.	Mid. 15 11·7	55 45·5

Distance at 6 hours, 44° 14′ 50″; at 9 hours, 45° 45′ 38″.

The variation of the compass.

The deflection of the magnetic needle from the true North, or, as it is usually called, the *variation of the compass*, is found at sea either by computing the true bearing of the sun from an observed altitude, the compass bearing being noted at the time of the observation; or, without taking an altitude, determining the true bearing of the sun when in the horizon, its compass bearing being observed at the same instant. The difference between the compass bearing and true bearing thus found is *the variation of the compass*.

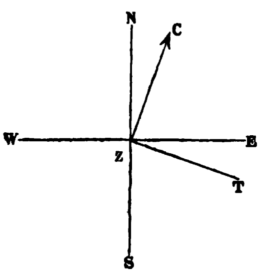
Sometimes it is necessary to correct the variation of the compass determined as above for the deviation of the compass itself, arising from the following local causes. The iron on board draws the needle to the east or west of the magnetic meridian, and this effect is greater or less on the needle according as the iron is distributed more or less unequally on different sides of the magnetic meridian. The deviation of the compass due to this cause is discovered, previously to the ship going to sea by swinging her round and noting the deflection of the needle from the magnetic

meridian on different points; a table is then formed similar to the one in p. 244, from which the correction of the compass for different positions of the ship's head may be readily found (see p. 16). The method of determining whether the variation of the compass is east or west will be best seen by means of the following examples.

EXAMPLES.

1. Suppose the true bearing of the sun was found by observation to be N. 100° 10′ E., when the compass bearing was N. 90° 42′ E.; required the variation of the compass, the ship's head being N.E.

Let N represent the true north point of the horizon, and N S the true meridian, measure (roughly) 100° 10′ from north towards east as the angle N Z T; then T represents the place of the sun when the observation was taken. From T measure back towards N the compass bearing 90° 42′, as T Z C; then Z C is the direction of the magnetic needle, and the angle N Z C is the variation of the compass, which is evidently easterly, since the compass north is to the east of the true north: hence in this example the variation is said to be east: thus,



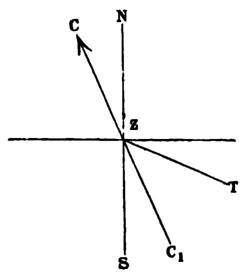
True bearing N Z T	. . .	N. 100° 10′ E.
Compass bearing C Z T	. .	N. 90 42 E.
Apparent variation.	. . .	9 28 E.

Now if the iron on board had no effect on the needle, this would be the true variation, but referring to the table it appears that the needle itself is drawn or deflected 10° to the east, in consequence of the disturbing effects of the

iron when the direction of the ship's head is N.E.: placing the 10° 0′ E., under the above 9° 28′ E., and subtracting we have the true variation of the compass corrected for local deviation: thus,

Observed variation	9° 28′ E.
Deviation	10 0 E.
True variation	0 32 W.

2. The true bearing of the sun was found by observation to be S. 60° 42′ E., when the compass bearing was S. 50° 10′ E.: required the variation of the compass, the ship's head being N.E.



Let N Z S represent as before the true meridian, draw Z T, making the angle S Z T, equal to 60° 42′ (roughly), then T represents the true place of the sun; from T measure back towards S, the compass bearing T Z C′, equal 50° 10′, then Z C′ is the direction of the magnetic needle, and the angle S Z C′ or N Z C is the observed variation of the compass, to be corrected for deviation (if any).

Thus,

True bearing	S. 60° 42′ E.
Compass bearing	S. 50 10 E.
Observed variation	10 32 W.
Deviation	10 0 E.
True variation	20 32 W.

For by the table it appears that the needle, by the effects of the iron, is drawn 10° to the eastward; if there had been no iron on board the needle would have been directed 10° to the westward of its observed place. Hence may be deduced the following rule to find the variation of the compass.

DEVIATION OF THE COMPASS OF H.M.S. ——.

(Caused by the local attraction of the Ship) for given positions of the Ship's head.

Direction of Ship's Head.	Deviation of Compass.	Direction of Ship's Head.	Deviation of Compass.
N.	2° 45′ E.	S.	3 0 W.
N. by E.	4 57	S. by W.	4 20
N.N.E.	7 30	S.S.W.	5 0
N.E. by N.	9 0	S.W. by S.	6 7
N.E.	10 0	S.W.	7 0
N.E. by E.	10 55	S.W. by W.	7 27
E.N.E.	10 40	W.S.W.	7 50
E. by N.	9 55	W. by S.	8 20
E.	8 50	W.	8 50
E. by S.	7 15	W. by N.	8 10
E.S.E.	5 35	W.N.W.	6 50
S.E. by E.	3 40	N.W. by W.	5 40
S.E.	1 50	N.W.	4 50
S.E. by S.	0 20 E.	N.W. by N.	3 20
S.S.E.	0 56 W.	N.N.W.	1 40 W.
S. by E.	2 20	N. by W.	1 10 E.

Rule LVII.

Given the true bearing and compass bearing and deviation, to find the variation of the compass.

1. Reckon the compass bearing and the true bearing from the same point, north or south.

2. Take the difference of the two bearings when measured towards the same point, but the sum when measured towards different points; the result is the apparent variation of the compass; east when the true bearing is to the right of the compass bearing, west if the true bearing is to the left of the compass; the observer being supposed to be placed in the centre of the compass, and looking towards the heavenly body.

NOTE.—The name of the variation, whether east or west, may also be readily found by making a figure similar to those in the preceding examples.

3. If there be no deviation to be allowed for local attraction, the above is the true variation.

4. *To correct for local deviation* (if any). Under the apparent variation just found, put the correction from the table of deviation, with the *opposite* letter to that given in the table.

5. When the names put down are alike add, putting the common letter to the result: if the names put down be unlike, subtract the less from the greater, putting to the remainder the name of the greater. The result will be the variation of the compass corrected for deviation, and therefore the true variation.

EXAMPLES.

The true bearing of the sun is N. 117° 32′ E., and compass bearing S. 71° 10′ E.: required the true variation. The ship's head being S.b.E., and therefore the deviation

of the compass 2° 20′ W. (see Table). The compass bearing reckoned from the same point as the true bearing is, N. 108° 50′ E.

True bearing	N. 117° 39′ E.	
Compass bearing	N. 108 50 E.	
Apparent variation . . .	8 49 E.	
Deviation	2 20 E.	
True variation	11 9 E.	

The true bearing is E. 10° N., when the compass bearing is E. 8° S.; required the true variation, the ship's head being S.W.

True bearing	E. 10° N.
Compass bearing	E. 8 S.
Apparent variation . . .	18 W.
Deviation	7 E.
True variation	11 W.

The true bearing is S. 80° W., when the compass bearing is N. 108° W.; required the true variation, the ship's head being S.W.b.W., and therefore the deviation by Table 7½° W.

True bearing	S. 80° W.
Compass bearing	S. 72 W.
Apparent variation . . .	8 E.
Deviation	7½ E.
True variation	15½ E.

(220.) The true bearing of the sun was N. 36° E., when the compass bearing was N. 24° E., the ship's head being W. ½ N.; required the variation of the compass.

Ans., 23½° E.

(221.) The true bearing was N. 110° 42′ W., when the

compass bearing was N. 90° 24′ W., the ship's head being S.S.W.; required the variation of the compass.

Ans., 15° 18′ W.

(222.) The true bearing was S. 48° 30′ W., when the compass bearing was N. 132° 33′ W., the ship's head being S.W.b.W.; required the variation of the compass.

Ans., 8° 30′ E.

(223.) The true bearing of the sun was E. 20° 20′ N., when the compass bearing was E. 32° 45′ N., the ship's head being W.; required the variation of the compass.

Ans., 21° 15′ E.

(224.) The true bearing of the sun was W. 12° 32′ S., the compass bearing was W. 2° 10′ N., the ship's head being W.b.S.; required the variation of the compass.

Ans., 6° 22′ W.

(225.) The true bearing of the sun was W. 30° 10′ N., the compass bearing was W. 20° 42′ N., the ship's head being N.b.E.; required the variation of the compass.

Ans., 4° 31′ E.

The variation of the compass is found at sea by either of the following problems.

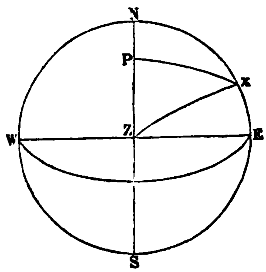
1. Given the latitude of the ship and the sun's declination when in the horizon, to find the bearing or amplitude.

2. Given the latitude of the ship, and the altitude of the sun and declination, to find the true bearing or azimuth.

3. Given the latitude and time at the ship and the sun's declination, to find the true bearing or azimuth.

The compass bearing being observed at the time of observation, the difference of compass and true bearing, that is, the variation of the compass, is readily found, by the preceding rules.

Variation by amplitude.



Let X be the heavenly body in the horizon. Then in the quadrantal triangle P Z X are given P Z = colat. P X = codecl. or polar distance, and Z X = 90°, to find the angle P Z X, and thence its complement X Z E the amplitude.

By trig. (p. 68.)

cos. P X = sin. P Z cos. P Z X.

or sin. decl. = cos. lat. sin. amplitude

∴ sin. amplitude = sin. decl. sec. lat.

Rule LVIII.

1. Get a Greenwich date.
2. Take out of the Nautical Almanac the sun's declination for this date.
3. Add together the log. sin. of the declination and log. secant of latitude; the sum, rejecting 10 in the index, is the log. sin of amplitude, which take from the tables.
4. If the body is rising, mark it east, if setting west: mark it also north or south according as the declination is north or south.
5. The result is the amplitude or true bearing.
6. Under the true bearing put the compass bearing, and determine the variation of the compass by the preceding rule.

EXAMPLE.

September 19, 1851, at $5^h\ 51^m$ A.M., mean time nearly, in latitude 47° 25′ N., and long. 72° 15′ W., the sun rose

by compass E. 12° 10′ N.; required the variation, the ship's head being E.b.S.

Ship, Sept. 18 . . .	17^h	51^m
Long. in time . . .	4	51 W.
Greenwich, Sept. 18 .	22	42

Sun's declination.									
18th	2°	0′	31″ N.	Sin. decl.				8·457103
19th	1	37	14 N.	Sec. lat.				0·169628
		23	17	Sin. ampl.				8·626731
·02419				True bearing . .	E.	2°	25′ N.	
·88823				Comp. bearing . .	E.	12	10 N.	
·91242		22	2	App. variation		9	45 E.	
Declination . .	1	38	29 N.	Deviation		7	15 W.	
				True variation . . .		2	30 E.	

(226.) May 6, 1846, at 5^h 30^m A.M., mean time nearly, in lat. 50° 48′ N., and long. 47° 12′ E., the sun rose by compass E. 2° 10′ S.; required the variation, the ship's head being S.b.W. Ans., 24° 21′ 30″ W.

(227.) Nov. 14, 1846, at 6^h 45^m P.M., mean time nearly, in lat. 32° 14′ S., and long. 100° E., the sun set by compass W. 15° 40′ S.; required the variation, the ship's head being N.E. Ans., 16° 1′ 30″ W.

(228.) January 10, 1846, at 6^h 58^m A.M., mean time nearly, in lat. 31° 56′ N. and long. 75° 30′ W., the sun rose by compass E. 30° 10′ S.; required the variation, the ship's head being N.E.b.E. Ans., 67° 14′ 45″ W.

(229.) March 21, 1846, at 6^h 0^m A.M., mean time nearly, in lat. 42° 13′ N., and long. 90° E., the sun rose by compass E. 11° 40′ S.; required the variation, the ship's head being W.b.S. Ans., 3° 20′ 15″ W.

(230.) March 31, 1850, at 6^h 0^m P.M., mean time nearly, in lat. 42° 13′ N. and long. 124° W., the sun set by compass W. 11° 30′ S.: required the variation, the ship's head being N. Ans., 14° 38′ 15″ E.

(281.) Dec. 4, 1851, at 7^h 50^m A.M., mean time nearly, in lat. 50° 40′ N., and long. 94° W., the sun rose by compass E. 10° 42′ S.; required the variation of the compass, the ship's head being N.E. Ans., 57° 20′ 45″ W.

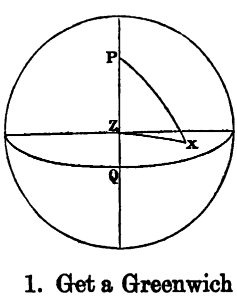
Elements from Nautical Almanac.

	Sun's declination.								
May 5 . .	16°	13′	22″ N. . . .	6 .	16°	30′	22″	N.
Nov. 14 .	18	13	22 S. . . .	15 .	18	28	54	S.
Jan. 10 .	21	58	29 S.					
March 20 .	0	11	37 S. . . .	21 .	0	12	5	N.
March 31 .	4	7	44 N. . . .	32 .	4	30	54	N.
Dec. 4 . .	22	13	9 S. . . .	5 .	22	21	7	S.

Rule LIX.

Variation by azimuth.

Given the altitude of the sun, and the compass bearing, to find the variation of the compass.



Let X be the place of the heavenly body when its compass bearing is observed: then in the triangle P Z X are given three sides, to find an angle, namely P Z = colat. P X = codecl. or polar distance, and Z X = zenith distance, to find the angle P Z X the true bearing or azimuth.

1. Get a Greenwich date.
2. Take out of the Nautical Almanac the sun's declination for this date, and also the sun's semidiameter.

3. Find the polar distance, by adding 90° to the declination, when the latitude and declination have different names, or by subtracting the declination from 90°, when the latitude and declination have the same names.

4. Correct the observed altitude for index correction, dip, semidiameter and correction in altitude, and thus get the true altitude.

5. Put down the latitude under the altitude and take their difference; under which put the polar distance; take the sum and difference.

6. To the log. secants of the two first terms in this form (omitting the tens in the index) add the halves of the log. haversines* of the two last, the result, rejecting 10 in the index, is the log. haversine of the true bearing or azimuth, which find from the table.

7. Mark the true bearing N. or S. according as the latitude is N. or S., mark it also E. or W., according as the heavenly body is E. or W. of the meridian.

8. With the true bearing thus found and the compass bearing, find the variation of the compass by Rule 57, p. 245.

EXAMPLE.

June 7, 1851, at 5^h 50^m A.M., mean time nearly, in lat. 50° 47′ N., and long. 99° 45′ W., when the sun bore by compass S. 92° 36′ E., the observed altitude of the sun's lower limb was 18° 35′ 20″, index correction + 3′ 10″, and the height of the eye above the level of the sea was 19 feet; required the variation, the ship's head being N.E.

Ship, June 6 . . .	17^h	50^m	
Long. in time . . .	6	39	W.
Greenwich, June 7 .	0	29	

* If the student have no table of haversines, the angle PZX, or true bearing, may be found in a similar manner to that pointed out in p. 171.

	Sun's declin.	Sun's semi.		Obs. alt.	
7th. . . .	22° 43′ 49″ N.	15′ 46″		18° 35′ 20″	
8th. . . .	22 49 34 N.		In cor.. . .	3 10	+
	5 45			18 38 30	
1·69597			Dip . . .	4 17	—
1·49560				18 34 13	
3·19157 .	0 7		Semi. . . .	15 46	
	22 43 56 N.		App. alt. . .	18 49 59	
	90		Cor. in alt. .	2 41	—
Pol. dist.. .	67 16 4		True alt.. .	18 47 18	

Latitude	50° 47′ 0″	. .	Sec. . .	0·199108
Altitude	18 47 18	. .	Sec. . .	0·023779
	31 59 42			
Polar dist.. . . .	67 16 4			
Sum.	99 15 46	. .	½ hav. .	4·881893
Difference. . . .	35 16 22	. .	½ hav. .	4·481384
				9·586164

True bearing . . .	N. 76° 46′ 30″ E.
Compass bearing. .	N. 87 24 0 E.
App. variation . .	10 37 30 W.
Deviation	10 0 0 W.
True variation . .	20 37 30 W.

When the ship is in harbour, or in any position where the sight of the horizon is intercepted by land, or obscured by fog, so that the altitude of the sun cannot be taken, the preceding methods are inapplicable. The following rule may then be used, in which it is supposed that the hour angle at the ship is known, or can be found by means of the chronometer, or the time at the place.

Rule LX.

Variation by azimuth (hour angle being known).

Given the hour angle at ship and the compass bearing, to find the variation of the compass.

Let x (fig. p. 250) be the place of the heavenly body when its compass bearing is observed: then in the triangle

P Z X are given two sides and the included angle, to find one of the remaining angles: namely, P Z = colat. P X = polar distance, and Z P X the hour angle, to find the angle P Z X, the true bearing or azimuth.

1. Get a Greenwich date.

2. Take out of the Nautical Almanac for this date, the equation of time and sun's declination.

3. Find the polar distance, by adding 90° to the declination, when the latitude and declination have different names, or by subtracting the declination from 90° when the latitude and declination have the same name.

4. Under the colatitude (found by subtracting the latitude from 90°) put the polar distance, take the sum and difference, and the half sum and half difference.

5. *To find the hour angle at ship.*

Correct the time shown by chronometer when the compass bearing was observed, for its error on Greenwich mean time, and thus get the mean time at Greenwich; to mean time apply the equation of time to obtain apparent time; under this put the longitude in time, adding if east and subtracting if west; the result will be ship apparent time, and also the hour angle if P.M.; but if A.M. at ship, subtract the apparent time from 24 hours, the remainder will then be the hour angle required.

6. Divide the hour angle by 2. Then under heads (1) and (2) put down the following quantities.

7. Under both (1) and (2) put log. cotangent of half hour angle.

Under (1) log. cosine } of half difference of polar distance
(2) log. sine. } and colatitude.
(1) log. sec. } of half sum of polar distance and
(2) log. cosec. } colatitude.

8. Add together the log. under (1) and (2) separately; and take out the angles corresponding to each as a log. tangent. Put one under the other, and take their sum, if the polar distance is greater than the colatitude, or their

difference if the polar distance is less than the colatitude; the result will be the true bearing of the sun at the time of observation.

9. Then proceed to find the variation as in Rule LVII.

EXAMPLE.

June 23, 1847, at 10^h 58^m A.M., mean time nearly, in lat. 50° 48′ and long. 1° 6′ W., when a chronometer showed 11^h 3^m 37^s, the bearing of the sun was observed to be N. 173° 10′ E., the error of the chronometer on Greenwich mean time being 0^m 54^s fast; required the variation.

June 22, at . .	22^h	58^m	
Long. in time .		4 +	
Gr. June 22 . .	23	2	
Colat. . . .	39°	12′	0″
Pol. dist. . .	66	33	0
Sum	105	45	0
Difference . .	27	21	0
½ sum . . .	52	52	30
½ difference .	13	40	30

Equation of time.		
22nd . .		1^m $29^s·72$
23rd . .		1 42·64 sub.
		12·92
	·01786	
	2·92283	
	2·94069	12·40
		1 42·12

Sun's declination.				
. . . .		23°	27′	21″ N.
. . . . . .		23	26	59 N.
				22
	·01786			
	2·69100			
	2·70886			21
		23	27	0
	Polar dist.	66	33	0

Time by chro. .	11^h	3^m	$37^s + 12^h$
Err. on Gr. m. t.		0	54 fast
Greenw. 22nd .	23	2	43
Eq. of time . .		1	42 sub.
App. time .	23	1	1
Long. in time .		4	24 W.
Ship app. time	22	56	37
	24		
Hour angle . .	1	3	23
½ hour angle .	0	31	41

(1)		(2)	
Cot. ½ h. ang. .	10·856573	. . . . .	10·856573
Cos. ½ diff. . .	9·987511	Sin. ½ diff. .	9·373674
Sec. ½ sum . .	10·219283	Cosec. ½ sum	10·098367
Tan.	11·063367	Tan. . . .	10·328614
	85° 3′ 30″		64° 51′ 45″.
	64 51 45		
True b. N.	149 55 15 E.		
Comp. b. N.	173 10 0 E.		
Variation.	23 14 45 W.		

(232.) April 27th, 1847, at 1^h 10^m P.M., mean time nearly, in lat. 50° 48′ N., and long. 1° 6′ W., when a chronometer showed 1^h 15^m 51^s, the bearing of the sun was observed to be S. 51° 55′ W., the error of the chronometer on Greenwich mean time being 1^m 18^s fast; required the variation. Ans., 23° 46′ W.

(233.) Dec. 14, 1847, at $10^h\ 22^m$ A.M., mean time nearly, in lat. 52° 10″ N., and long. 1° 30′ W., when a chronometer showed $10^h\ 30^m\ 48^s$, the bearing of the sun was observed to be N. 179° 20′ E., the error of the chronometer on Greenwich mean time being $3^m\ 38^s$ fast; required the variation.

Ans.. 21° 23′ 15″ W.

(234.) Dec. 14, 1847, at $1^h\ 55^m$ P.M., mean time nearly, in lat. 48° 50′ N., and long. 1° 30′ W., when a chronometer showed $1^h\ 59^m\ 55^s$, the bearing of the sun was observed to be S. 51° 40′ W., the error of the chronometer on Greenwich mean time being $0^m\ 5^s$ fast; required the variation. Ans., 22° 25′ 15″ W.

(235.) Dec. 14, 1848, at $11^h\ 11^m$ A.M., mean time nearly, in lat. 39° 40′ N., and long. 0° 40′ E., when a chronometer showed $11^h\ 19^m\ 43^s$, the bearing of the sun was observed to be N. 167° 50′ E., the error of the chronometer on Greenwich mean time being $3^m\ 38^s$ fast; required the variation.

Ans., 2° 50′ 13″ E.

(236.) March 7, 1844, at $9^h\ 59^m$ A.M., mean time nearly, in lat. 49° 48′ N., and long. 1° 10′ E., when a chronometer showed $10^h\ 24^m\ 8^s$, the bearing of the sun was observed to be N. 164° 51′ 40″ E., the error of the chronometer on Greenwich mean time being fast $20^m\ 48^s$; required the variation. Ans., 20° 26′ 40″ W.

(237.) May 26, 1851, at $9^h\ 48^m$ A.M., mean time nearly, in lat. 50° 48′ N., and long. 1° 6′ W., when a chronometer showed $9^h\ 47^m\ 37^s$, the bearing of the sun was observed to be S. 31° 7′ E., the error of the chronometer on Greenwich mean time being $3^m\ 17^s$ fast; required the variation.

Ans., 23° 35′ 15″ W.

Elements from Nautical Almanac.

	Sun's declination.	Equation of time.
April 27	13° 43′ 22″ N.	27 . . 2^m $24^s{\cdot}56$ add
„ 28	14 2 25 N.	28 . . 2 34·29
Dec. 13	23 8 42 S.	13 . . 5 43·66 add
„ 14	23 12 38 S.	14 . . 5 15·08
Dec. 14	23 12 38 S.	14 . . 5 15·08 add
„ 15	23 16 7 S.	15 . . 4 46·23
Dec. 13	23 11 43 S.	13 . . 5 23·23 add
„ 14	23 15 18 S.	14 . . 4 54·52
March 6	5 30 13 S.	6 . . 11 25·48 sub.
„ 7	5 6 55 S.	7 . . 11 10·81
May 25	20 53 42 N.	25 . . 3 25·40 add
„ 26	21 4 25 N.	26 . . 3 19·55

We will conclude this treatise of the practical part of Nautical Astronomy with a series of examination papers given at the Royal Naval College, to candidates passing for lieutenants' and masters' commissions in the Royal Navy. They are also required to have some knowledge of arithmetic, algebra, and trigonometry: the papers now set on these subjects contain examples in arithmetic as far as vulgar and decimal fractions; in algebra, as far as simple equations, inclusive; and in trigonometry, the examples and problems are similar to those contained in Part I. of the author's Trigonometry.

ROYAL NAVAL COLLEGE EXAMINATION PAPERS.

Questions.—No. I.

1. Required the course and distance from A to B.

Lat. A . .	56° 35′ S.	Long. A . .	2° 15′ E.
B . .	51 10 S.	B . .	3 10 W.

2. Required the course and distance from A to B.

Lat. A . .	61 10 N.	Long. A . .	8° 40′ E.
B . .	61 10 N.	B . .	15 10 E.

3. A ship bore from me S. ¼ E., and a current ran in the intermediate space S.W. ¾ W. 4½ miles an hour; how must I steer a boat to fetch the ship, supposing I can pull 6 miles an hour in still water?*

4. On May 8, 1835, at noon, a point of land in lat. 48° 10′ N., and long. 2° 2′ W., bore by compass E. by S. ½ S. distant 20 miles, (variation 3¼ E.); afterwards sailed as by the following log account; required the latitude and longitude in at noon, on May 9, 1835.

Hours.	Knots.	$\frac{1}{10}$ths.	Course.	Wind.	Leeway	
1	3	0	N.N.W. ½ W.	N.E.	2¼	P.M.
2	3	2				
3	3	4				
4	2	7				
5	2	4	E. by S. ¾ S.	Do.	3	
6	2	4				Variation 3¼ E.
7	3	6				
8	3	2				
9	3	2				
10	4	0	E.S.E.	South.	2¾	
11	4	2				
12	4	5				
						Remarks in H.M.S. May 9, 1835.
1	5	2	N.N.E. ¼ E.	N.W.	1¾	A.M.
2	6	2				
3	7	2				
4	7	5				
5	8	0				
6	5	2	W.S.W.	Do.	3¼	
7	6	2				
8	6	4				
9	5	4				
10	6	2				
11	6	3				
12	7	0				

* The current sailing question is now omitted in the Navigation Paper at the Royal Naval College. It sometimes appears among the questions in trigonometry

5. What bright star will pass the meridian of Canton in China the first after mean midnight on June 15, 1835, and how far N. or S. of the zenith?

6. June 15, 1835, in long. 100° 32′ E., the observed meridian altitude of the sun's lower limb was 20° 15′ 40″ (zenith S. of the sun), the index correction was + 2′ 50″, and the height of the eye above the sea was 14 feet; required the latitude.

7. April 23, 1835, at 9^h P.M., mean time nearly, in long. 5° 10′ W., the observed meridian altitude of the moon's lower limb was 38° 40′ 45″, (zenith N. of the moon), the index correction was — 2′ 50″, and the height of eye above the sea was 20 feet; required the latitude.

8. June 18, 1835, the observed meridian altitude of the star α Scorpii (Antares) was 20° 10′ 50″ (zenith north of the star), the index correction was + 4′ 50″, and the height of the eye above the sea was 18 feet; required the latitude.

9. June 12, 1835, the observed meridian altitude under the S. Pole of α^2 Crucis was 6° 40′ 10″, the index correction was + 3′ 40″, and the height of the eye above the sea was 18 feet; required the latitude.

10. December 10, 1835, at 2^h 10^m A.M., mean time nearly, in long. 76° 12′ E., the observed altitude of α Ursæ Minoris (Polaris), was 47° 50′ 25″ the index correction was — 4′ 10″, and the height of the eye above the sea was 13 feet; required the latitude.

11. September 16, 1835, observed the following double altitude of the sun.

Mean time nearly.	Chronometer.	Obs. alt. sun's L. L.	True bearing.
10^h 40^m A.M.	10^h 22^m 36^s	40° 34′ 30″	S. b. E. ½ E.
11 40 A.M.	11 22 45	42 12 0	S. ½ E.

The run of the ship in the interval was E. b. N. 12 miles, the index correction was + 3′ 50″, and the height of the eye above the sea was 18 feet; required the true latitude, the latitude by account being 51° N., and the longitude 50° 10′ W.

12. March 2, 1835, at 7^h 44^m A.M., mean time nearly, in lat. 44° 25′ N., and long. by account 58° E., a chronometer showed 5^h 10^m $42^s \cdot 5$, and the observed altitude of α Arietis was 30° 10′ 40″ W., of meridian, the index correction was + 4′ 20″, and the height of eye above the sea was 18 feet; required the true longitude.

February 24, at Greenwich mean noon, the chronometer was fast on Greenwich mean time 1^h 11^m 22^s, and its daily rate was $2^s \cdot 2$ losing.

13. Sept. 3, 1835, at 9^h 10^m P.M., mean time nearly, in lat. 30° 10′ N., and long. by account 91° 5′ E., the following lunar observation was taken.

Obs. alt. α Arietis E. of meridian.	Obs. alt. moon's L.L.	Obs. dist. F. L.
10° 15′ 40″	36° 12′ 30″	99° 27′ 50″
+ 1 40	— 1 10	— 0 30

The height of the eye above the sea was 12 feet; required the true longitude.

14. July 5, 1835, at 7^h 0^m P.M., mean time, in lat. 50° 53′ N., and long. 120° 10′ E., the compass bearing of the sun was W. 10° 15′ N., and the observed altitude of its lower limb was 9° 40′ 0″, the index correction was + 3′ 50″, and the height of the eye above the sea was 18 feet; required the variation of the compass.

15. On Dec. 20, 1835, at 4^h 30^m P.M., mean time nearly, in lat. 41° 12′ N., and long. 110° 45′ E., the sun set by compass S.W.; required the variation.

16. Required the time of high water at A., on June 10, 1835, A.M. and P.M.

Change tide . 3^h 40^m A.M. app. time. Long. A. . 65° W.

NOTE.—In this and the following examination papers the compass is supposed to have no deviation arising from local attraction. In the Mercantile Navy the correction for deviation is not generally attended to: but in Her Majesty's Service all ships are now swung previously to their going to sea, and a table of deviations constructed similar to the one in p. 244 for the correction of courses, &c.

Elements from Nautical Almanac and Answers.

1. N. 30° 29′ 15″ W. 377·2 miles.
2. E. 188·1 miles.
3. S.E. ¼ S.
4. Corrected courses N.W. ¾ N. 20′ departure course, N.b.W.½W. 12′·3; S. 14′·8; S.E.b.E.½E. 12′·7; E.¾N. 34′·1; W.S.W. 42′·7. Lat. in 48° 6′ N., long. 2° 18′ W.
5. Right ascension mean sun on June 14, at Greenwich mean noon, $5^h\ 32^m\ 11^s{\cdot}85$. γ Draconis, 28° 23′ N. of zenith.
6. Sun's declination on June 14, at Greenwich mean noon, 23° 15′ 26″ N.; on June 15, 23° 18′ 24″ N., semidiameter 15′ 46″. Lat. 46° 14′ 16″ S.
7. Moon's declination on April 22, at 21^h Greenwich mean time, 11° 35′ 58″ S., at 22^h..11° 23′ 53″ S.; moon's horizontal semidiameter April 22, at Greenwich mean midnight, 15′ 4″·1, April 23, at Greenwich mean noon, 15′ 0″; corresponding horizontal parallax 55′ 17″·8 and 55′ 2″·8. Lat. 38° 57′ 50″ N.
8. Declination α Scorpii (Antares) 26° 3′ 34″ S. Lat. 43° 47′ 34″ N.
9. Declination α^2 Crucis 62° 11′ 25″ S. Lat. 34° 20′ 28″ S.
10. Right ascension mean sun on June 16, at Greenwich mean noon, $17^h\ 10^m\ 2^s{\cdot}21$; lat. 47° 51′ N.
11. Sun's declination on September 15, at Greenwich mean noon, 2° 49′ 54″ N., on September 16, 2° 26′ 42″ N., semidiameter 15′ 56″. Lat. 50° 20′ N.
12. Right ascension mean sun March 2, at Greenwich mean noon, $22^h\ 38^m\ 13^s{\cdot}55$. Right ascension α Arietis, $1^h\ 57^m\ 51^s{\cdot}5$; declination α Arietis 22° 40′ 42″ N. Hour angle $4^h\ 37^m\ 23^s$ W. Long. 59° 12′ 15″ E.
13. Right ascension mean sun September 3, at Greenwich mean noon, $10^h\ 47^m\ 36^s{\cdot}37$. Right ascension α Arietis $1^h\ 57^m\ 55^s{\cdot}3$; declination α Arietis 22° 40′ 56″ N. Horizontal semidiameter moon September 3, at Greenwich mean noon

15′ 51″·7, at Greenwich mean midnight 15′ 48″·3; corresponding horizontal parallax 58′ 12″·5, and 57′ 59″·9. True distance 98° 59′ 4″; distance from Nautical Almanac at III. 99° 2′ 53″, at VI. 97° 22′ 28″. Hour angle 17^h 54^m 56^s W. Long. 89° 28′ 30″ E.

14. Sun's declination on July 4, at Greenwich mean noon 22° 56′ 37″ N., on July 5, 22° 51′ 24″ N.; semidiameter 15′ 45″. True bearing N. 65° 41′ W. Variation 14° 4′ E.

15. Sun's declination on December 19, at Greenwich mean noon 23° 25′ 35″ S.; on December 20, 23° 26′ 46″ S. True bearing W. 31° 55′ 30″ S. Variation 13° 4′ 30″ E.

16. Moon's Greenwich meridian passage June 10, 12^h 2^m. June 9, 11^h 0^m; moon's semidiameter 16′ 36″. Equation of time 1^m S. to apparent time. High water 3^h 2^m A.M., and 3^h 33^m P.M.

NOTE.—The right ascension of mean sun is found in the Nautical Almanac in page II. of each month under the heading of "Sidereal Time."

Questions.—No. II.

1. Required the course and distance from A to B.

Lat. A . .	40° 25′ N.	Long. A . .	2° 10′ E.
B . .	35 32 N.	B . .	1 40 W.

2. Required the course and distance from A to B.

Lat. A . .	50° 48′ N.	Long. A . .	100° E.
B . .	50 48 N.	B . .	101 E.

3. A ship bore from me W. ¾ N., and a current run in the intermediate space N.N.W. 4½ miles an hour; how must I steer to fetch the ship, supposing I can pull in still water 5½ miles an hour?

4. May 10, 1837, at noon, a point of land in lat. 38° 17′ N. and long. 56° 19′ W., bore by compass W. b. S. ¼ S. distant

17½ miles (variation of compass 2¾ E.); afterwards sailed as by the following log account; required the latitude and longitude in, May 11, at noon.

Hours.	Knots.	$\frac{1}{10}$ths.	Course.	Wind.	Leeway	
1	5	4	S.S.E.	E.	2¼	P.M.
2	5	6				
3	4	9				
4	4	8				
5	4	8	S.S.W.½W.	W.	2¾	
6	4	7				Variation 2¾ E.
7	5	3				
8	5	2				
9	5	1				
10	6	0	W.S.W.	S.½W.	2½	
11	6	4				
12	6	8				
						H.M.S. May 11, 1836.
1	6	7				A.M.
2	5	9				
3	5	8	W.½N.	N.N.E.	0	
4	4	6				During the last 7 hours a current set the ship 3 miles an hour, N.N.E. by compass.
5	4	8				
6		9				
7	4	8				
8	3	7				
9	3	6	E.	S.S.E.	2½	
10	3	4				
11	3	5				
12	2	9				

5. What bright star will pass the meridian of Greenwich the first after 10^h P.M., on October 20, 1837, and how far N. or S. of the zenith?

6. October 19, 1837, in longitude 88° 49′ E., the observed meridian altitude of the sun's lower limb was 58° 37′ 56″ (zenith N. of the sun); the index correction was + 8′ 38″, and the height of the eye above the sea was 17 feet; required the latitude.

7. August 10, 1837, at 6^h 40^m P.M. mean time, in long. 50° 17′ E., the observed meridian altitude of the moon's lower limb was 45° 47′ 39″ (zenith N. of the moon), the index correction was − 3′ 18″, and the height of the eye above the sea was 24 feet; required the latitude.

8. June 3, 1837, the observed meridian altitude of the

star α Canis Majoris was 43° 29′ 47″ (zenith S. of the star), the index correction was − 3′ 14″, and the height of the eye above the sea was 16 feet; required the latitude.

9. February 18, 1837, the observed meridian altitude of the star α Ursæ Majoris under the North Pole was 53° 28′ 47″, the index correction was − 3′ 49″, and the height of the eye above the sea was 18 feet; required the latitude.

10. February 9, 1837, at $10^h\ 20^m$ P.M. mean time, in long. 85° 32′ W., the observed altitude of α Ursæ Minoris (Polaris) was 50° 25′ 30″, the index correction was − 4′ 10″, and the height of the eye above the sea was 15 feet; required the latitude.

11. June 9, 1837, the following double altitude of the sun was observed.

Mean time nearly.	Chronometer.	Obs. alt. sun's L. L.	True bearing.
$1^h\ 3^m$ P.M.	$1^h\ 10^m\ 50^s$	52° 5′ 40″	S.S.W.
7 6 P.M.	7 12 48	14 57 30	W.N.W.

The run of the ship in the interval was N.N.E. 5 miles, the index correction was − 1′ 20″, and the height of the eye above the sea was 17 feet; required the true latitude at the second observation; the latitude by account being 59° N. and longitude 47° 18′ E.

12. August 25, 1837, at $9^h\ 45^m$ P.M. in lat. 60° 2′ N. and longitude by account 59° 15′ E. when a chronometer No. 10, showed $5^h\ 42^m\ 16^s$, the observed altitude of the star α Andromedæ was 39° 32′ 28″ E. of the meridian; the index correction was + 5′ 17″, and the height of the eye above the sea was 15 feet; required the true longitude.

On May 15, 1837, at Greenwich mean noon, No. 10 was *slow* on Greenwich mean time $8^m\ 40^s{\cdot}5$, and its daily rate was $7^s{\cdot}8$ *losing*.

13. May 14, 1837, at $2^h\ 20^m$ P.M. mean time nearly, in lat. 50° 48′ N., and longitude by account 60° 52′ E., the following lunar observation was taken.

	Obs. alt. sun's L.L.	Moon's L. L.	Obs. dist. N. L.
	46° 48′ 7″	45° 47′ 38″	108° 58′ 45″
Index cor.	+ 3 10	— 1 12	+ 2 18

The height of the eye above the sea was 10 feet; required the true longitude.

14. May 20, 1837, at 4^h 47^m A.M. mean time nearly, in lat. 18° 42′ S. and long. 160° E., the sun rose by compass E. 21° 18′ 30″ N.; required the variation.

15. March 7, 1837, at 2^h 50^m P.M. mean time nearly, in lat. 51° 10′ N. and long. 86° E., the compass bearing of the sun was S. 74° 42′ W.; and at the same time the observed altitude of the sun's lower limb was 21° 40′ 45″, the index correction was — 2′ 18″, and the height of the eye above the sea was 14 feet; required the variation.

16. Required the time of high water at A on August 27, 1837, A.M. and P.M.

Change tide at A . 5^h 18^m P.M. app. time. Long. A . 93° E.

Elements from Nautical Almanac and Answers.

1. S. 31° 43′ 30″ W. 344′·5.
2. E. 37′·9.
3. S.W. ¾ W.
4. Corrected courses E. b. S. ½ S. 17′·5; S.W. b. S. 20′·7; S.S.W. ½ W. 25′·1; N.W. ¾ W. 31′·8; N.W. ¾ W. 29′·6; E. ½ S. 13′·4; N.E. ¾ E. 21′. Lat. in 38° 20′ 36″ N. Long. in 56° 52′ W.
5. α Andromedæ 23° 17′ 10″ S. of zenith.
6. Sun's declination on October 18, at Greenwich mean noon, 9° 39′ 16″ S.; on October 19, 10° 1′ 3″ S.; semidiameter 16′ 5″. Lat. 21° 6′ 29″ N.
7. Moon's declination on August 10, at 3^h, Greenwich mean time, 23° 15′ 12″ S.; on August 10, at 4^h . . 23° 24′ 37″ S.; moon's horizontal semidiameter on August 10, at Greenwich mean noon, 15′ 43″·8; on August 10, at Greenwich mean midnight, 15′ 51″·3; correction horizontal parallax 7′ 43″·5 and 58′ 11″·0 Lat. 20° 7′ 1″ N.

8. Declination of α Canis Majoris 16° 29′ 49″. S. Lat. 63° 8′ 14″ S.

9. Declination of α Ursæ Majoris 62° 37′ 42″ N. Lat. 80° 42′ 22″ N.

10. Right ascension mean sun, on February 9, at Greenwich mean noon $21^h\ 17^m\ 28^s.28$. Lat. 50° 33′ N.

11. Sun's declination on June 8, at Greenwich mean noon, 22° 51′ 58″ N.; on June 9, 22° 57′ 10″ N.; semidiameter 15′ 46″. Arc (1) 81° 40′ 15″, Arc (2) 68° 32′ 15″, Arc (3) 38° 4′ 0″. Lat. 60° 11′ 51″ N.

12. Right ascension mean sun on August 25, at Greenwich mean noon $10^h\ 14^m\ 9^s.84$; declination of α Andromedæ 28° 11′ 40″ N. Hour angle $20^h\ 4^m\ 23^s$ W. Long. 56° 15′ 0″.

13. Sun's declination on May 13, at Greenwich mean noon, 18° 24′ 17″ N.; on May 14, 18° 38′ 55″ N.; correction equation of time $3^m\ 55^s.3$ A and $3^m\ 55^s.9$ A; moon's horizontal semidiameter on May 13, at Greenwich mean midnight, 14′ 55″.2; on May 14, at Greenwich mean noon, 15′ 59″; corresponding horizontal parallax, 54′ 45″.1 and 54′ 58″.9. True distance, 108° 37′ 59″; distance at XXI, 108° 4′ 49″; distance on 14, at Greenwich mean noon, 109° 28′ 44″. Hour angle $2^h\ 24^m\ 5^s$. Long. 62° 15′ E.

14. Sun's declination on May 19, at Greenwich mean noon, 19° 47′ 14″ N.; on May 20, 19° 59′ 54″ N. True bearing E. 20° 59′ 45″ N. Variation, 0° 18′ 45″ E.

15. Sun's declination on March 6, at Greenwich mean noon, 5° 37′ 27″ S.; on March 7, 5° 14′ 8″ S.; semidiameter, 16′ 8″. True bearing N. 130° 56′ 30″ W. Variation, 25° 38′ 30″ W.

16. Moon's Greenwich meridian passage on August 27, $22^h\ 8^m.7$; August 26, $21^h\ 19^m.9$; moon's semidiameter, 14′ 45″. Equation of time $1^m\ 19^s$ from mean time. High water $2^h\ 18^m$ A.M. and $2^h\ 42^m$ P.M.

NOTE.—The right ascension of mean sun is found in the Nautical Almanac in page II. of each month under the heading of "Sidereal Time."

Questions.—No. III.

1. Required the course and distance from A to B.

Lat. A . .	70° 15′ S.	Long. A . .	3° 10′ W.
B . .	75 20 S.	B . .	2 15 E.

2. How many miles are there in 10° of longitude in the latitude of Portsmouth?

3. A ship bore from me S.S.W. $\frac{3}{4}$ W., and a current ran in the intermediate space S.S.E. $\frac{3}{4}$ E., $7\frac{3}{4}$ miles an hour; how must I steer a boat to fetch the ship, supposing I can pull in still water $10\frac{1}{2}$ miles an hour?

4. March 4, 1837, at noon, a point of land in lat. 50° 48′ N., and long. 1° 6′ W., bore by compass N.N.E. $\frac{1}{2}$ E., distant 15 miles, (variation $2\frac{1}{4}$ W.), afterwards sailed as by the following log account; required the latitude and longitude in, on March 5, at noon.

Hours.	Knots.	$\frac{1}{10}$ths.	Course.	Wind.	Leeway	
1	3	5	N.N.W. $\frac{1}{2}$ W.	N.E.	$1\frac{3}{4}$	P.M.
2	4	1				
3	4	3				
4	2	7	E S.E.	Do.	2	
5	3	0				
6	3	2				Variation $2\frac{1}{4}$ W.
7	4	0				
8	5	6	S. $\frac{3}{4}$ E.	E.S.E.	$2\frac{1}{2}$	
9	5	2				
10	5	5				
11	4	5				
12	4	6				
						Remarks in H.M.S. Mar. 5, 1837.
1	4	7	N.E. $\frac{1}{2}$ N.	Do.	$1\frac{1}{2}$	A.M.
2	4	2				
3	4	4				
4	3	7				
5	3	2				A current set the ship N.E. the last 6 hours at the rate of $3\frac{1}{4}$ miles per hour.
6	3	5	W. $\frac{1}{2}$ N.	S.S.W.	$1\frac{1}{4}$	
7	4	2				
8	3	6				
9	3	4				
10	9	5	N. by E.	South.	0	
11	10	2				
12	10	3				

5. At what time will the star α Lyræ pass the meridian of Portsmouth on May 11, 1837, and how far N. or S. of the zenith?

6. March 8, 1837, in long. 89° 48′ E., the observed meridian altitude of the sun's lower limb was 51° 49′ 30″, zenith north of the sun, the index correction was — 3′ 17″, and the height of the eye above the sea 15 feet; required the latitude.

7. March 16, 1837, at $8^h\ 2^m$ P.M., mean time nearly, in long. 110° E., the observed meridian altitude of the moon's lower limb was 48° 47′ 36″, zenith north of the moon, the index correction was — 2′ 47″, and the height of the eye above the sea was 13 feet; required the latitude.

8. July 7, 1837, the observed meridian altitude of the star α Cygni was 53° 29′ 38″, zenith north of the star, the index correction was — 5′ 12″, and the height of the eye above the sea was 16 feet; required the latitude.

9. Oct. 16, 1837, the observed meridian altitude of the star α Ursæ Majoris under the North Pole was 5° 26′ 10″, the index correction was — 2′ 10″, and the height of the eye above the sea was 17 feet; required the latitude.

10. Sept. 10, 1837, at $3^h\ 42^m$ A.M., mean time, in long. 83° 14′ E., the observed altitude of α Ursæ Minoris was 39° 47′ 48″, the index correction was + 3′ 45″, and the height of the eye above the sea was 17 feet; required the latitude.

11. April 10, 1837, the following double altitude of the sun was observed.

Mean time nearly.	Chronometer.	Obs. alt. sun's L. L.	True bearing.
$10^h\ 14^m$ A.M.	$10^h\ 9^m\ 40^s$	41° 15′ 45″	S.E.
11 47 A.M.	11 43 28	46 43 12	S. by E.

The run of the ship in the interval was N.W. 6 miles, the index correction was – 4′ 24″, and the height of the eye above the sea was 20 feet; required the true latitude at the second observation, the latitude by account being 51° N., and the longitude 1° 6′ W.

12. May 10, 1837, at 3^h 10^m P.M., mean time, in lat. 48° 12′ N., and long. by account 45° 10′ E., when a chronometer showed 0^h 10^m 42^s, the observed altitude of the sun's lower limb was 37° 20′ 10″, the index correction was + 3′ 10″, and the height of the eye above the sea was 18 feet; required the true longitude.

On May 1, 1837, at Greenwich mean moon, the chronometer was fast on Greenwich mean time 9^m 50^s, and its daily rate was $3^s{\cdot}2$ gaining.

13. January 16, 1837, at 3^h 4^m P.M., mean time nearly, in lat. 50° 50′ N., and long. by account 65° E., the following lunar observation was taken.

	Obs. alt. sun's L. L.	Obs. alt. moon's L. L.	Obs. dist. N. L.
	8° 32′ 20″	15° 42′ 30″	121° 10′ 30″
Index cor.	+ 1 10	+ 5 47	— 5 47

The height of the eye above the sea was 16 feet: required the true longitude.

14. May 18, 1837, at 4^h 50^m A.M., mean time nearly, in lat. 18° 45′ S., and long. 99° 18′ E., the sun rose by compass S. 80° 12′ E.; required the variation.

15. March 7, 1837, at 9^h 10^m A.M., mean time nearly, in lat. 51° 10′ N., and long. 89° 12′ E., the compass bearing of the sun was S. 74° 50′ E., and at the same time the observed altitude of the sun's lower limb was 21° 40′ 43″, the index correction was — 2′ 18″, and the height of the eye above the sea was 14 feet; required the variation.

16. Required the time of high water at A. on March 10, 1837, A.M. and P.M.

Change tide at A . 6^h 45^m P.M. app. time. Long. A . 98° E.

Elements from Nautical Almanac and Answers.

1. S. 17° 23′ E., 319′·4.
2. 379′·2.
3. W. b. S. $\frac{3}{4}$ S.

4. Corrected courses S. $\frac{1}{4}$ W. 15′, or W.S.W. $\frac{1}{4}$ W.; departure course W. b. N. $\frac{1}{2}$ N. 11′·9; E.S.E. $\frac{1}{4}$ E. 12·9; S. $\frac{3}{4}$ W. 25′·4; N. 20′·2; W. $\frac{1}{2}$ S. 14′·7; N. b. W. $\frac{1}{4}$ W. 30′. N.b. E. $\frac{3}{4}$ E. 21′. Latitude in 51° 14′ 54″ N. Longitude in 1° 35′ 24″ W.

5. At $15^h\ 12^m\ 42^s$: 12° 10′ 11″ S. of zenith.

6. Sun's declination on March 7, at Greenwich mean noon, 5° 14′ 8″ S.; on March 8, 4° 50′ 46″ S.; semidiameter 16′ 7″. Latitude 33° 5′ 44″ N.

7. Moon's declination on March 16, at 0^h, 26° 48′ 39″ N.; at 1^h, 26° 44′ 20″ N. moon's horizontal semidiameter on March 16, at Greenwich mean noon 14′ 45″·1; on March 16, at Greenwich mean midnight 14′ 44″·9; corrected horizontal parallax 54′ 8″·1 and 54′ 7″·4. Latitude 67° 14′ 35″ N.

8. Declination α Cygni 44° 41′ 59″ N. Lat. 81° 22′ 12″ N.

9. Declination of α Ursæ Majoris 62° 37′ 28″ N. Latitude 32° 33′ 1″ N.

10. Right ascension mean sun, on Sept. 9, at Greenwich mean noon, $11^h\ 13^m\ 18^s{\cdot}15$. Latitude 38° 24′ N.

11. Sun's declination on April 9, at Greenwich mean noon 7° 36′ 29″ N., on April 10, 7° 58′ 43″ N. Semidiameter 15′ 58″. Arc (1) 23° 13′ 15″, Arc (2) 88° 17′ 30″, Arc (3) 65° 27′ 0″. Latitude 51° 0′ 47″ N.

12. Sun's declination on May 10, at Greenwich mean noon 17° 38′ 34″; on May 11, 17° 54′ 7″ N., correct equation of time $3^m\ 50^s{\cdot}2$ S., and $3^m\ 52^s.5$ E.; semidiameter 15′ 51″. Hour angle $3^h\ 31^m\ 21^s$. Longitude 51° 47′ E.

13. Sun's declination on January 15, at Greenwich mean noon, 21° 6′ 42″ S., on January 16, 20° 55′ 23″ S., correct equation of time $9^m\ 49^s{\cdot}1$ A and $10^m\ 9^s{\cdot}8$ A.; moon's horizontal semidiameter on January 15, at Greenwich mean midnight 15′ 2″·1, on January 16, at Greenwich mean noon, 14′ 58″·0; corresponding horizontal parallax 55′ 10″·6 and 54′ 55″·3. True distance 121° 20 29″; distance at XXI., 120° 32′ 5″, at XXIV., 121° 55′ 25″. Hour angle $2^h\ 54^m\ 7^s$. Longitude 64° 55′ 45″ E.

14. Sun's declination on May 17, at Greenwich mean noon, 19° 20′ 53″ N., on May 18, 19° 34′ 14″ N., true bearing E. 20° 34′ 45″ N. Variation 30° 22′ 45″ W.

15. Sun's declination on March 6; at Greenwich mean noon, 5° 37′ 27″ S., on March 7, 5° 14′ 8″ S., semidiameter 16′ 8″, true bearing N. 131° 9′ 15″ E., variation 25° 59′ 15″ E.

16. Moon's Greenwich meridian passage on March 10, $3^h\ 13^m$ mean time on March 9, $2^h\ 26^m$; moon's semidiameter 15′ 37″, equation of time 11^m S. from mean time, high water $9^h\ 1^m$ P.M. and $8^h\ 37^m$ A.M.

NOTE.—The right ascension of mean sun is found in the Nautical Almanac in page II. of each month under the heading of "Sidereal Time."

Questions.—No. IV.

1. Required the course and distance from A to B.

Lat. A	. .	60°	25′ S.	Long. A	. .	35°	22′ E.
B	. .	64	12 S.	B	. .	30	10 E.

2. Sailed from Ushant due west 492·5 miles; required the latitude and longitude in.

3. A ship bore from me N.E. ¼ E. and a current set in the intermediate space N. ¼ W., 5 miles an hour; how must I steer a boat to fetch the ship, supposing I can pull in still water 7¼ miles an hour?

4. May 1, 1835, at noon, a point of land in latitude 51° 10′ S., and long. 3° 15′ E., bore by compass S.S.W ½ W. distant 25 miles, variation 2¾ E., afterwards sailed as by the following log account; required the latitude and longitude in.

Hours.	Knots.	$\frac{1}{10}$ths.	Course.	Wind.	Leeway	
1	3	2	S.S.E. ½ E.	S.W.	2¼	A.M.
2	3	4				
3	3	5				
4	4	0				
5	4	2				
6	4	3	W.N.W.	Do.	2½	Variation 2¾ E.
7	4	4				
8	4	5				
9	5	2				
10	5	6				
11	5	7				
12	6	1				
						H.M.S. May 2, 1835.
1	7	2	South.	West.	¼	P.M.
2	7	1				
3	7	3				
4	8	3				
5	8	4				
6	7	5	S.E. ¾ E.	W. by S. ¾ S.	1½	A current set the ship N.W. ½ W. 20 miles.
7	7	2				
8	7	1				
9	6	7				
10	6	5				
11	6	3				
12	6	0				

5. What bright star will pass the meridian of the Land's End the first after $6^h\ 42^m$ A.M. mean time on August 17, 1835, and how far N. or S. of the zenith?

6. August 18, 1835, in long. 110° 32′ E., the observed meridian altitude of the sun's lower limb was 50° 25′ 10″, zenith N. of the sun, the index correction was — 2′ 50″; and the height of the eye above the sea was 15 feet; required the latitude.

7. August 18, 1835, at $8^h\ 0^m$ A.M., mean time nearly, in long. 92° 10′ W., the observed meridian altitude of the moon's lower limb was 26° 42′ 10″, zenith S. of the moon, the index correction was — 3′ 40″, and the height of the eye above the sea was 14 feet; required the latitude.

8. December 7, 1835, the observed meridian altitude of the fixed star *a* Arietis was 40° 25′ 10″, zenith N. of the star, the index correction was – 2′ 10″, and the height of the eye above the sea was 18 feet; required the latitude.

12. December 7, 1835, the observed meridian altitude

of α Ursæ Majoris, under the N. Pole was 11° 10′ 10″, the index correction was + 3′ 20″, and the height of the eye above the sea was 19 feet; required the latitude.

10. December 7, 1835, at 1^h 20^m A.M., in long. 78° 30′ E., the observed altitude of α Ursæ Minoris (Polaris), was 50° 40′ 15″, the index correction was − 5′ 10″ and the height of the eye above the sea was 12 feet; required the latitude.

11. July 30, 1835, observed the following double altitude of the sun.

Mean time nearly.	Chronometer.	Obs. alt. sun's L. L.	True bearing.
11^h 58^m A.M.	0^h 0^m 10^s	57° 29′ 45″	S. 3° E.
0 4 P.M.	0 6 17	57 29 30	S. 3° 20′ W.

The run of the ship was none, dip none, the index correction was + 0′ 30″; required the true latitude, the latitude by account being 51° N., and the longitude 1° W.

12. May 14, 1835, at 9^h 30^m A.M., in lat. 50° 48′ N., and long. by account 2° W., a chronometer showed 9^h 26^m 18^s, and the observed altitude of the sun's lower limb was 46° 48′ 7″, the index correction was 3′ 10″ + and the height of the eye above the sea was 10 feet; required the true longitude.

May 1, 1835, at Greenwich mean noon, the chronometer was slow on Greenwich mean time 4^m 2^s and its daily rate was $3^s{\cdot}5$ losing.

13. September 3, 1835, at 7^h 32^m P.M., mean time nearly, in lat. 30° 10′ N., and long. by account 36° 10′ W., the following lunar observation was taken.

	Obs. alt. α Pegasi (Markab) E. of meridian.	Obs. alt. moon's L. L.	Obs. dist. F. L.
	9° 50′ 40″	18° 10′ 50″	55° 46′ 20″
Index cor.	− 1 10	+ 1 30	− 0 30

The height of the eye above the sea was 15 feet; required the true longitude.

14. August 23, 1835, at 7^h 0^m P.M., mean time nearly, in lat. 50° 48′ N., and long. by account 140° 25′ E., the sun set by compass W. 5° 10′ S.; required the variation.

15. August 23, 1835, at $5^h 50^m$ A.M., mean time nearly, in lat. 51° 10′ N., and long. 135° 40′ W., the sun bearing by compass was S. 22° 10′ E., and the observed altitude of its lower limb was 7° 40′ 50″, the index correction was — 2′ 50″, and the height of eye above the sea was 15 feet; required the variation.

16. Required the mean time of high water at A, on Aug. 2, 1835, A.M. and P.M.

Change tide . $3^h 40^m$ app. time. Long. A . 70° W.

Elements from Nautical Almanac and Answers.

1. S. 32° 32′ 15″ W. 269′·3.
2. Lat. in 48° 28′ N. Long. in 17° 25′ 48″ W.
3. E. b. N.
4. Corrected courses N.E. b. E. ¼ E. 25′ departure course, S.S.E. 18′·3; N. ¾ W. 35′·8; S.S.W. ½ W. 38′·3; S.E. ½ S. 47′·3; N. b. W. ¾ W. 20′. Lat. in 51° 29′ 54″ S. Long. in 4° 0′ 18″ E.
5. α Tauri 33° 54′ S. of zenith.
6. Sun's declination on August 17, at Greenwich mean noon, 13° 35′ 52″ N., on August 18, 13° 16′ 40″·3 N., semidiameter 15′ 49″. Latitude, 52° 48′ 51″ N.
7. Moon's declination on August 18, at 2^h Greenwich mean time, 24° 12′ 3″ N., at 3^h, 24° 17′ 6″ N., moon's horizon semidiameter on August 18, at Greenwich mean noon, 14′ 51″·6, at midn. 14′ 54″·5, corresponding horizontal parallax 54′ 31″·8 and 54′ 42″·7. Latitude 38° 10′ 36″ S.
8. Declination of α Arietis 22° 41′ 5″ N. Latitude 72° 23′ 24″ N.
9. Declination of α Ursæ Majoris 62° 37′ 57″ N. Latitude 38° 26′ 29″ N.
10. Right ascension mean sun on Dec. 6, at Greenwich mean noon, $16^h 58^m 12^s{\cdot}52$. Latitude 50° 16′ N.

11. Sun's declination on July 31, at Greenwich mean noon, 18° 39′ 18″ N., on August 1, 18° 24′ 47″ N., semidiameter 15′ 47″. Latitude 50° 53′ 30″ N.

12. Sun's declination on May 13, at Greenwich mean noon, 18° 16′ 32″ N.; on May 14, 18° 31′ 19″ N; corresponding equation of time $3^m\ 45^s{\cdot}9$ S., and $3^m\ 55^s{\cdot}9$ S.; semidiameter, 15′ 50″; hour angle, $21^h\ 36^m\ 45^s$ W. Longitude 0° 26′ 0″ E.

13. Right ascension mean sun September 3, at Greenwich mean noon, $10^h\ 47^m\ 36^s{\cdot}37$; right ascension α Pegasi $22^h\ 56^m\ 35^s{\cdot}2$; declination 14° 19′ 23″ N.; moon's horizontal semidiameter on September 3, at Greenwich mean noon, 15′ 51″·7, at midnight, 15′ 48″·3; corresponding horizontal parallax 58′ 12″·5, and 57′ 59″·9; true distance 55° 32′ 23″; distance from Nautical Almanac at VI. 56° 49′ 7″, at IX. 55° 19′ 51″; hour angle $18^h\ 11^m\ 57^s$ W. Longitude 33° 48′ 0″ W.

14. Sun's declination on August 22, at Greenwich mean noon, 11° 57′ 49″ N., on August 23, 11° 37′ 37″ N.; true bearing W. 18° 38′ 45″ N. Variation 23° 48′ 45″ E.

15. Sun's declination on August 23, at Greenwich mean noon, 11° 37′ 37″ N.; on August 24, 11° 17′ 14″ N.; semidiameter 15′ 51″; true bearing N. 81° 6′ 30″ E. Variation 6° 43′ 30″ W.

16. Moon's Greenwich meridian passage August 2, $6^h\ 38^m$; August 1, $5^h\ 46^m$; semidiameter 16′ 10″; equation of time 6^m S. from mean time. High water $9^h\ 12^m$ A.M., and $9^h\ 38^m$ P.M.

NOTE.—The right ascension of mean sun is found in the Nautical Almanac in page II. of each month under the heading of "Sidereal Time."

Questions.—No. V.

1. Required the course and distance from A to B.

Lat. A . .	65° 25′ S.	Long. A . .	3° 28′ W.
B . .	73 42 S.	B . .	4 2 E.

2. Required the course and distance from C to D.

Lat. C . .	70° 15′ N.	Long. C . .	15° 25′ E.
D . .	70 15 N.	D . .	20 25 E.

3. A point of land bore from me S.S.W. ½ W., and a current set in the intermediate space S.S.E. 4¾ miles an hour; how must I steer a boat to fetch the point of land, supposing I can pull in still water 8½ miles an hour?

4. October 23, 1837, at noon, a point of land in latitude 34° 28′ S., and longitude 18° 28′ E, bore by compass N.W. distant 10 miles (variation of compass 2¼ W.), afterwards sailed as by the following log account; required the latitude and longitude in, on October 24, at noon.

Hours.	Knots.	$\frac{1}{10}$ths.	Course.	Wind.	Leeway	
1	5	4	N. by E. ½ E.	N.W. ½ W.	2¼	
2	5	2				
3	5	8				
4	6	1				
5	6	5	S.S.W.	W.N.W.	¼	
6	7	3				Variation 2¼ W.
7	7	0				
8	7	2				
9	6	8	N.W. by W.	S.E.	0	
10	6	5				
11	6	1				
12	5	8	S. by W. ¾ W.	S.E. ½ E.	2¼	
						—— Oct. 24, 1837.
1	6	0				
2	6	5				
3	6	8				
4	6	4	N.N.E.	N.W.	2	
5	6	0				
6	6	5				A current set the ship the last 5 hours N.W. 2 miles an hour.
7	6	8				
8	7	2	N.W.	East.	0	
9	7	6				
10	7	9				
11	8	1				
12	8	5				

5. At what time will the star α Aquilæ (Altair) pass the meridian of the Land's End, on December 8, 1837, and how far N. or S. of the zenith?

6. December 10, 1837, in long. 55° 20′ E., the observed meridian altitude of the sun's lower limb was 25° 52′ 5″ (zenith N.); the index correction was − 2′ 10″, and the height of the eye above the sea was 17 feet; required the latitude.

7. August 10, 1837, at $6^h\ 40^m$ P.M. mean time nearly, in long. 50° 17′ E., the observed meridian altitude of the moon's lower limb was 45° 47′ 39″ (zenith N. of the moon); the index correction was − 3′ 18″, and the height of the eye above the sea was 24 feet; required the latitude.

8. October 15, 1837, the observed meridian altitude of α Aquilæ was 50° 25′ 30″ (zenith N.); the index correction was − 3′ 20″, and the height of the eye above the sea was 13 feet; required the latitude.

9. October 16, 1837, the observed meridian altitude of α Ursæ Majoris (Dubhe) under the North Pole was 5° 26′ 10″; the index correction was − 2′ 10″, and the height of the eye above the sea was 17 feet; required the latitude.

10. March 17, 1837, at $9^h\ 43^m$ P.M. mean time, in long. 93° 14′ W., the observed altitude of α Ursæ Minoris (Polaris) was 32° 49′ 14″; the index correction was + 7′ 49″, and the height of the eye above the sea was 12 feet; required the latitude.

11. March 14, 1837, the following double altitude of the sun was observed.

Mean time nearly.	Chronometer.	Obs. alt. sun's L. L.	True bearing.
$1^h\ 5^m$ P.M.	$8^h\ 2^m\ 25^s$	41° 20′ 45″	S.S.W. ½ W.
5 6 P.M.	12 3 30	7 29 30	W. by S. ½ S.

The run of the ship in the interval was N.E. 18 miles; the index correction was − 3′ 20″, and the height of the eye above the sea was 23 feet; required the true latitude at the second observation; the latitude by account being 45° N. and the long. 50° 20′ W.

12. February 10, 1837, at 9^h 20^m P.M. mean time nearly, in lat. 28° 20′ N. and longitude by account 31° 2′ W., a chronometer showed 11^h 16^m 25^s, and the observed altitude of the star α Leonis (Regulus) was 41° 55′ 10″ E. of the meridian; the index correction was + 1′ 20″, and the height of the eye above the sea was 25 feet; required the true longitude.

On February 1, 1837, at Greenwich mean noon, the chronometer was *fast* on Greenwich mean time 5^m $20^s{\cdot}6$, and its daily rate was $2^s{\cdot}7$ *losing*.

13. April 27, 1837, at 2^h 30^m A.M. mean time nearly, in lat. 45° 20′ N., and longitude by account 46° W., the following lunar observation was taken.

	Obs. alt. α Virginis W. of meridian.	Obs. alt. moon's L.L.	Obs. dist. F. L.
	16° 30′ 50″	15° 38′ 56″	98° 2′ 40″
Index cor.	+ 2 20	+ 5 52	+ 1 5

The height of the eye above the sea was 12 feet; required the true longitude.

14. June 15, 1837, at 8^h 10^m P.M. mean time, in lat. 50° 48′ N., and long. 73° 19′ E., the sun set by compass W. 30° 29′ N.; required the variation.

15. June 15, 1837, at 9^h 39^m A.M. mean time nearly, in lat. 50° 48′ N., and long. 99° 29′ E., the compass bearing of the sun was S. 38° 19′ 50″ E., and the observed altitude of the sun's lower limb at the time was 49° 58′ 37″; the index correction was + 10′ 43″, and the height of the eye above the sea was 12 feet; required the variation.

16. Required the time of high water at A on February 17, 1837, A.M. and P.M.

Change tide at A . 11^h 42^m P.M. app. time. Long. A . 2° W.

Elements from Nautical Almanac and Answers.

1. S. 17° 19′ 15″ E. 520′·6.

2. E. 101′·3.

3. S.W. $\frac{3}{4}$ W.

4. Corrected courses E. b. S. $\frac{3}{4}$ S. 10′ departure course; N.b.E.$\frac{1}{2}$E. 22′·5; S.$\frac{1}{2}$E. 28′; W.$\frac{3}{4}$N. 19′·4; S.b.W.$\frac{3}{4}$W. 25′·1; N.b.E.$\frac{3}{4}$E. 25′·7; W.b.N.$\frac{3}{4}$N. 39′·3; W.b.N.$\frac{3}{4}$N. 10′. Latitude in 34° 17′ 54″ S. Longitude in 17° 32′ E.

5. At 2h 34m and 41° 37′ S. of zenith.

6. Sun's declination on December 9, at Greenwich mean noon, 22° 51′ 14″ S.; on December 10, 22° 56′ 49″ S.; semidiameter 16′ 16″. Lat. 41° 3′ 48″ N.

7. Moon's declination on August 10, at 3h . . 23° 15′ 12″ S.; at 4h . . 23° 24′ 37″ S.; moon's horizontal semidiameter on August 10, at Greenwich mean noon, 15′ 43″·8; on August 10, at Greenwich mean midnight, 15′ 51″·3; corresponding horizontal parallax, 57′ 43″·5 and 58′ 11″·0. Lat. 20° 7′ 1″ N.

8. Declination of *a* Aquilæ 8° 26′ 42″ N. Lat. 48° 8′ 53″ N.

9. Declination of *a* Ursæ Majoris 62° 37′ 28″ N. Lat. 32° 33′ 1″ N.

10. Right ascension mean sun, on March 17, at Greenwich mean noon, 23h 39m 24s·25. Lat. 33° 47′ N.

11. Sun's declination on March 14, at Greenwich mean noon, 2° 29′ 26″ S.; on March 15, 2° 5′ 46″ S.; semidiameter, 16′ 6″. Arc (1) 60° 12′ 45″, Arc (2) 91° 26′ 30″, Arc (3) 46° 23′ 30″. Lat. 43° 59′ 23″ N.

12. Sun's right ascension on February 10, at Greenwich mean noon, 21h 21m 24s·83; right ascension *a* Leonis, 9h 59m 43s; declination, 12° 45′ 40″ N. Hour angle, 20h 43m 40s. Long. 27° 50′ 45″ W.

13. Right ascension mean sun on April 26, at Greenwich mean noon, 2h 17m 6s·41; right ascension of *a* Virginis, 13h 16m 38s; declination, 10° 18′ 40″ S.; moon's horizontal semidiameter on April 26, at Greenwich mean midnight,

16′ 8″·3; on April 27, at Greenwich mean noon, 16′ 8″·4; corresponding horizontal parallax, 59′ 13″·5 and 59′ 13″·7. True distance, 97° 30′ 29″; distance at XV, 96° 0′ 34″; at XVIII, 97° 46′ 47″. Hour angle, 3^h 34^m 26^s W. Long. 45° 19′ 30″ W.

14. Sun's declination on June 15, Greenwich mean noon, 23° 19′ 50″ N.; on June 16, 23° 22′ 11″ N. True bearing W. 38° 48′ 45″ N. Variation, 8° 19′ 45″ E.

15. Sun's declination on June 14, at Greenwich mean noon, 23° 17′ 5″ N.; on June 15, 23° 19′ 50″ N.; semidiameter, 15′ 46″. True bearing N. 119° 53′ E. Variation, 21° 47′ 10″ W.

16. Moon's meridian passage on February 17, 10^h $21^m·9$; on February 16, 9^h $32^m·2$; semidiameter, 14′ 43″. Equation of time, 14^m S. from mean time. High water, 9^h 32^m A.M. and 9^h 57^m P.M.

NOTE.—The right ascension of mean sun is found in the Nautical Almanac in page II. of each month, under the heading of "Sidereal Time."

APPENDIX.

Rule for finding the time of high water.

The Change Tide, upon which the rule is made to depend, is that tide which takes place P.M. on the day the moon changes, or is at full. The time of high water at change of the moon is given at different places in apparent time; and indeed cannot be generally expressed in mean time. If the tide be given A.M. on that day as the *change tide,* it should be reduced to P.M. by adding 18 minutes, which may be considered as an average difference on that day.

Rule.

Given the apparent time of change tide, and the longitude of the place: to find the mean time of high water A.M. *and* P.M.

Take out of the Nautical Almanac the moon's meridian passage on the given day, and also on the preceding day: also the moon's semidiameter, and equation of time for the given day (roughly).

Under heads (1) (2) and (3) (see Examples), put down the following quantities.

Under (1) the time of moon's meridian passage on proposed day, as found in the Nautical Almanac.

„ (3) the meridian passage on preceding day.

„ (2) put down half the sum of these times. (See Examples.)

Correct quantity under (1) by table (k), p. 5 of Inman's Tables, by entering with longitude of place at top, and difference of the times under (1) and (3) at the side: thus find the time of moon's meridian passage at the place.

Take out the correction from table (l), p. 5, and place it under (1). This correction is found as follows: Enter the table at top with moon's semidiameter, and at the side with meridian passage under (1), corrected by equation of time to nearest minute, so as to reduce the time of moon's meridian passage (which is given in mean time) to apparent time.

Apply the correction thus found with its proper sign, and to the result add the given apparent time of change tide.

1. When the quantity under (1) is *less* than 12 hours.

The time thus found is the mean time of high water P.M. for the proposed day. (See Example 1.)

2. When the quantity under (1) is *greater* than 12 hours, and less than 24 hours.

Work as described above with the meridian passage under (2). Then, if the result is greater than 12 hours, reject 12 hours: the remainder is mean time of high water on the proposed day, P.M. (See Example 3.) But if the result be less than 12 hours, it will be the mean time of high water A.M. on the proposed day. (See Example 5.)

3. When the quantity under (1) is greater than 24 hours.

Work as described above, with the meridian passage under (3). Then, if the result be greater than 24 hours, reject 24 hours: the remainder will be the mean time of high water P.M. on the proposed day. (See Example 7.) But if the result be less than 24 hours, and greater than 12 hours, reject 12 hours: the remainder will be the mean time of high water A.M. on the proposed day. (See Example 11.)

To find the next time of high water A.M. *or* P.M.

If the time of high water found as above is the P.M. time, subtract therefrom the difference between meridian passages under (1) and (2): the remainder will be the mean time of high water A.M. on the proposed day.

If the time of high water is the A.M. time, add thereto the difference between the meridian passages under (1) and (2), and the sum will be the mean time of high water P.M. on the proposed day.

If it be necessary to add 12 hours before this difference can be subtracted, in that case the remainder will be the mean time of high water P.M. on the preceding day: there will be no high water A.M. on the proposed day. And if in adding the difference the sum be greater than 12 hours, this sum, rejecting 12 hours, will be the mean time of high water A.M. on the following day: there will be no high water P.M. on the proposed day.

EXAMPLES.

(1.) Find the time of high water on January 3, 1857. Change tide $2^h\ 10^m$ P.M. apparent time. Long. 50° W.

Moon's mer. pass. Jan. 3, $6^h\ 7^m$ ☾ semi 16′ 10″
„ „ 2, 5 19 Eq. of time $4^m\ 54^s-$

$$\begin{array}{r} 48 \\ \hline 24 \end{array}$$

(1)

$$\begin{array}{l} 6^h\ 7^m \\ \quad\ 7+ \\ \hline 6\ 14 \\ 1\ \ 0- \\ \hline 5\ 14 \\ 2\ 10+ \\ \hline 7\ 24 \text{ P.M.} \\ \quad 24- \\ \hline 7\ \ 0 \text{ A.M.} \end{array} \Bigg\} \text{January 3.}$$

(2)

$$\begin{array}{r} 24 \\ \hline 5\ 43 \end{array}$$

(3)

$5^h\ 19^m$

(2.) Find the time of high water on May 28, 1857. Change tide $5^h\ 30^m$ P.M. app. time; long. 75° E.; moon's mer. pass. on 28, $4^h\ 51^m$; on 27, $3^h\ 58^m$; ☾ semi 15′ 41″; eq. of time $3^m\ 1^s+$.

Ans. $9^h\ 1^m$ P.M.; $8^h\ 35^m$ A.M.

(3.) Find the time of high water on January 12, 1857. Change tide $1^h\ 30^m$ P.M. app. time; long. 60° E.

Moon's mer. pass. Jan. 12,	$14^h\ 28^m$	☾ semi 15′ 22″
„ „ 11,	13 40	Eq. of time $8^m\ 42^s-$
	48	
	24	

(1)	(2)	(3)
$14^h\ 28^m$	14 4	$13^h\ 40^m$
8−	8−	
14 20	13 56	
0 38−	0 29−	
13 42	13 27	
1 30+	1 30+	
15 12	14 57	
Greater than 12 hours.	2 57 P.M.	January 12.
	24−	
	2 33 A.M.	

(4.) Find the time of high water on June 8, 1857. Change tide $4^h\ 20^m$ P.M. app. time; long. 40° W.; moon's mer. pass. on June 8, $13^h\ 4^m$; on 7, $12^h\ 10^m$; ☾ semi 14′ 58″; eq. of time $1^m\ 17^s+$. Ans. $5^h\ 5^m$ P.M.; $4^h\ 38^m$ A.M.

(5.) Find the time of high water on February 5, 1857. Change tide $2^h\ 10^m$ P.M. app. time; long. 70° E.

Moon's mer. pass. Feb. 5,	$9^h\ 37^m$	☾ semi 15′ 49″
„ „ 4,	8 37	Eq. of time $14^m\ 19^s-$
	60	
	30	
	9 7	

(1)	(2)	(3)
$9^h\ 37^m$	$9^h\ 7^m$	$8^h\ 37^m$
12 −	12 −	
9 25	8 55	
0 27 +	0 20 +	
9 52	9 15	
2 10 +	2 10 +	
12 2	11 25 A.M.	February 5.
Greater than 12 hours.	30 +	
	11 55 P.M.	

(6.) Find the time of high water on September 26, 1857. Change tide $6^h\ 40^m$ P.M. app. time; long. 20° W.; moon's mer. pass. on Sept. 26, $6^h\ 13^m$; on 25, $5^h\ 20^m$; ☾ semi 14′ 59″; eq. of time $8^m\ 43^s$+. Ans. $11^h\ 32^m$ A.M.; $11^h\ 58^m$ P.M.

(7.) Find the time of high water on January 24, 1857. Change tide $5^h\ 0^m$ P.M. app. time; long. 30° W.

Moon's mer. pass. Jan. 24, $23^h\ 51^m$ ☾ semi 15′ 41″
" " 23, 22 54 Eq. of time $12^m\ 27^s$−

57

28

(1)	(2)	(3)
$23^h\ 51^m$	23 23	$22^h\ 54^m$
5 +		5 +
23 56		22 59
0 11 +		0 20 +
24 7		23 19
5 0 +		5 0 +
29 7		28 19
Greater than 24 hours.		4 19 P.M.
	January 24	28 −
		3 51 A.M.

(8.) Find the time of high water on June 19, 1857. Change tide $3^h\ 40^m$ P.M. app. time; long. 120° E.; moon's mer. pass. on June 19, $22^h\ 28^m$; on 18, $21^h\ 27^m$; ☾ semi 16′ 32″; eq. of time $0^m\ 58^s-$. Ans. $1^h\ 19^m$ P.M.; $0^h\ 49^m$ A.M.

(9.) Find the time of high water on March 4, 1857. Change tide $5^h\ 0^m$ P.M. app. time; long. 45° W.

Moon's mer. pass. March 4,	$7^h\ 32^m$	☾ semi 15′ 48″
" " 3,	6 31	Eq. of time $11^m\ 54^s-$
	61	
	30	

(1)	(2)	(3)
$7^h\ 32^m$	7 2	$6^h\ 31^m$
7+	7+	
7 39	7 9	
0 15−	0 34−	
7 24	6 35	
5 0+	5 0+	
12 24	11 35 A.M. March 4; no P.M. tide.	
Greater than 12 hours.	30+	
	0 5 A.M. March 5.	

(10.) Find the time of high water on July 30, 1857. Change tide $5^h\ 30^m$ P.M. app. time; long. 90° W.; moon's mer. pass. on July 30, $7^h\ 6^m$; on 29, $6^h\ 20^m$; ☾ semi 14′ 49″; eq. of time $6^m\ 8^s-$. Ans. $11^h\ 48^m$ A.M., July 30; no P.M. tide.

(11.) Find the time of high water on March 21, 1857. Change tide $3^h\ 0^m$ P.M. app. time; long. 100° W.

Moon's mer. pass. March 21,	$21^h\ 11^m$	☾ semi 15′ 50″
" " 20,	20 17	Eq. of time $7^m\ 16^s-$
	54	
	27	
	20 44	

(1)		(2)	(3)	
21^h	11^m	20^h 44^m	20^h	17^m
	14+			14+
21	25		20	31
0	27+		0	13+
21	52		20	44
3	0+		3	0+
24	52	March 21, A.M.	11	44; no P.M. tide.
			11	27+
Greater than 24 hours.		March 22, A.M.	0	11

(12.) Find the time of high water on November 12, 1857. Change tide 2^h 40^m P.M. app. time; long. 70° W.; moon's mer. pass. on November 12, 21^h 24^m; on 11th, 20^h 43^m; ☾ semi 15′ 0″; eq. of time 15^m 40^s+.

Ans. 11^h 49^m A.M., November 12; no P.M. tide.

THE END.

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